Leverage Choice and Credit Spread Dynamics when Managers Risk Shift

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Abstract

We provide new insights that link compensation structure terms to credit spreads by modeling the dynamic risk choice of a risk-averse manager paid with performance insensitive pay (cash) and performance sensitive pay (stock). The model predicts that credit spreads are increasing in the ratio of cash-to-stock. When the manager has discretion to choose debt levels, a tradeoff between tax benefits and utility cost from ex-post asset substitution arises. The resulting optimal initial leverage is high with safe (risky) debt when cash-to-stock ratios are low (high), while moderate cash-to-stock ratios are associated with low initial leverage. In an empirical exercise using a large cross-section of 608 US based corporate credit default swaps (CDS) covering 2001-2006, we find strong evidence that CDS rates are high for CEOs with high salaries relative to stock and option holdings.

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1 Introduction

CEO compensation terms typically include a portion of pay that is performance sensitive and a portion of pay that is performance insensitive. These two components can be readily rationalized in a contracting setting (e.g., Holmstrom (1982)). Performance sensitive pay links a manager's value enhancing actions to his wealth to provide motivation to act in the interest of the firm. Performance sensitive pay is risky, however, and in contracts with a risk-averse manager risk-sharing motives give rise to a role for the performance insensitive component. Such compensation structures define the manager's objective and it is natural to expect it to affect a manager's choice of firm risk and, as a consequence, the dynamics of the firm's security prices. Although aspects of these linkages have been explored empirically, the structural modeling of the impact of CEO pay structure on risk choice, capital structure and the pricing of financial securities remains relatively unexplored.

In this paper, we undertake a step toward theoretically linking CEO compensation to security price dynamics, with a primary focus on credit spreads. We develop a structural model of a firm run by a manager whose pay includes a performance sensitive component (stock) and a performance insensitive component (cash). We assume that the manager is risk averse and can dynamically alter firm asset risk. The main finding of the model is that credit spreads should relate positively to the ratio of cash-to-stock. As this measure of compensation structure has not been empirically explored in the credit spread literature. we undertake an empirical exercise for a sample of US firms where we identify cash as CEO salary, and stock as the CEO's effective stock holdings due to stock and option ownership.¹ We find evidence that cash-to-stock is a statistically significant and economically important positive predictor of credit default swap (CDS) rates.² This result provides a possible reconciliation of the mixed results from regressions of credit spreads on managerial stock ownership that omit proxies for cash,³ since our model predicts that ownership is informative for spreads only when scaled relative to cash pay levels. Our empirical evidence and a calibrated version of our theoretical model also suggest that the quantitative failing of structural model of default sensible securities can be addressed by incorporating plausible agency costs.⁴

¹Following Core and Guay (2002), we define effective stock holdings as the sensitivity of the dollar change in the CEO's portfolio of stock and options to a 1% change in stock price.

 $^{^{2}}$ In a CDS agreement, the buyer of default protection pays a fixed quarterly amount until either the maturity date of the contract or the date a which a default event occurs. If default occurs, the buyer receives from the seller the difference between the market value and face value of the bond adjusted for accrued interest. The amount of the quarterly payment is referred to as the CDS rate, typically quoted in basis points relative to the contract's notional amount. For a more detailed discussion of CDS contracts see, for example, Duffie (1999).

³Anderson *et al* (2002) find that high managerial ownership predicts low spreads while Ortiz-Molina (2006) finds that high managerial ownership predicts high credit spreads. Bagnani et al (1994) find that spreads are increasing in ownership when the CEO has a small stake in the company and that spreads are decreasing in ownership when the CEO has a high stake in the company.

⁴When early structural models from the prior literature are calibrated to match equity volatility they predict credit spreads that are counter-factually low, especially when leverage ratios are low (see, for example, Jones, Mason, and Rosenfeld (1984) and Eom, Helwege and Huang (2003)). Our model relates to a later

As in Merton (1974), we begin our analysis by considering firms with a positive level of debt and cast our structural model of credit spreads in continuous time. The manager's stock compensation has no value in bankrupt states and, as a result, his pay is convex in firm value. Under the assumption that compensation terms cannot be undone by trading on personal account, the cash-and-stock pay structure leads the manager to dynamically alter firm volatility. We develop analytical formulas to describe the evolution of underlying firm asset value, stock prices, and bond prices. Our analysis singles out the cash-to-stock ratio in CEO pay as the key determinant of managerial risk-shifting behavior.⁵ Our characterization of credit spreads allows a robust and parameter independent comparative static analysis that shows, all else equal, high cash-to-stock leads to high spreads. This prediction is opposite to the prediction from previous models in which the manager is risk-neutral (e.g., Brander and Poitevin (1992) and John and John (1993)). Cash pay has no impact on risk neutral managers' choices in those settings, while adding convex pay in the form of stock induces more risk taking. The behavior of a risk averse manager in the presence of cash and stock pay can be very different, however, a point made clear in Ross (2004). In our setting, risk averse managers compensated with risky stock alone will act to avoid bankruptcy and make debt safe. Adding cash compensation provides managers with a minimum pay level in all states and gives rise to behavior that transfers wealth from bondholders in bankrupt states to stockholders in solvent states. In our model, therefore, it is the level of cash relative to stock compensation that provides the incentive to risk shift and inflates credit spreads.

Having characterized firm behavior with arbitrary debt levels, we also address the possibility that credit spreads are jointly determined with leverage ratios. We extend our structural model by assuming that, in addition to firm risk, managers also chose the level of debt as in Leland (1998). Debt has a direct impact on firm value since we assume that it gives rise to tax benefits and default costs. Managers, therefore, face a trade-off when choosing leverage. Their wealth increases when leverage increases, due to the effect of tax shields on the value of their stock holdings. Firm risk also increases, because of ex-post asset substitution, and the manager's optimal leverage choice balances the wealth benefit of debt against the utility cost associated with higher risk.⁶ A numerical exploration of

⁶This non-standard trade-off is similar in spirit to that of Berk, Stanton and Zechner (2006). They show that the human capital risk associated with bankruptcy can give rise to an implicit cost of debt that is sufficiently large to offset tax benefits even at low debt levels. Unlike in our model, their debt is optimally riskless, so that credit spreads are always zero, and there is no role for operating flexibility.

literature that modifies assumptions regarding the economics underlying the firm to address the quantitative failings of earlier structural models, such as Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Collin-Dufresne and Goldstein (2001), Francois and Morellec (2004), Morellec (2004), and David (2007).

 $^{^{5}}$ It is interesting to note that many firms explicitly mandate a maximum cash-to-stock ratio through "target ownership plans" (see, e.g., Core and Larcker (2002)). Boeing provides a specific example in their 2006 proxy statement: "In order to ensure continual alignment with our shareholders, we have stock ownership requirements for NEOs, other officers and senior executives. The ownership requirements have been in place since 1998 and are based on a multiple of base salary tied to executive grade. The stock ownership guidelines require that, within a three-year period, executives should attain and maintain an investment position in Boeing stock and stock units of the following: CEO - 6x base salary, Executive Vice Presidents and Senior Vice Presidents - 4x base salary, Vice Presidents - 1x or 2x base salary based on executive grade ..."

the debt choice model shows a non-monotonic relationship between the choice of initial leverage and cash-to-stock, where high leverage ratios are associated with very high or very low cash-to-stock and low leverage ratios are associated with moderate cash-to-stock. The analysis shows also that the positive association between cash-to-stock and credit spreads remains intact when leverage is endogenous, with safe debt issued when cash-to-stock is low and risky debt issued when cash-to-stock is high. This finding highlights that leverage ratios alone may not be sufficient to determine the value of new debt and that managerial pay structure can provide information that is of first-order importance for evaluating credit spreads.

To empirically assess the main prediction of a positive relationship between credit spreads and cash-to-stock, we make use of a large panel dataset of CDS rates for as many as 608 US based firms. The CDS market offers an attractive setting in which to examine the relative performance of default-sensitive security pricing models. First, changes in CDS rates are mainly driven by changes in default probabilities and default risk premia (see Berndt etal. (2005), and Longstaff, Mithal, and Neis (2005)). This is in contrast to corporate bond yields, for which liquidity and tax treatment play a significant role (see Chen, Lesmond, and Wei (2007), Delianedis and Geske (1998), Elton, Gruber, Agrawal, and Mann (2001), and Huang and Huang (2003)). Second, structural models relate company fundamentals (e.g. current market leverage ratios) to the price of default risk, and CDS rates have been shown to efficiently convey default-relevant information (see Blanco, Brennan and Marsh (2005), Acharya and Johnson (2006), and Norden and Weber (2004)). Finally, CDS markets are themselves economically important, with notional single-name CDS contract amounts at 2006 year end of roughly US 6.6 trillion (British Bankers Association). To the best of our knowledge, no previous work has examined the relationship between CDS rates and compensation terms.

Guided by our theoretical model, we consider a number of regression specifications that explain firm-year CDS rates with annually observable CEO pay terms from Compustat's ExecuComp database. When using CEO salary to proxy for performance insensitive pay and CEO effective stock ownership to proxy for performance sensitive pay, we confirm the model's prediction of a significantly positive coefficient on cash-to-stock. Our empirical model suggests that CEO compensation terms provide economically important information on the cross-section of CDS rates, even after controlling for the traditional structural determinants of spreads such as leverage and stock volatility. To illustrate this point, our estimated coefficients indicate that an increase in the cash-to-stock ratio from the 10^{th} to 90^{th} percentile can produce CDS rates that are up to 21% higher.

To ensure robustness of our empirical finding we control for potential endogeneity in our regression by including numerous economically motivated controls, industry fixed effects, and year fixed effects. The finding is also robust to a broader interpretation of the cash variable. Cash in our model is a component of managerial wealth that cannot be traded and is unrelated to firm performance. We consider two additional economic forces that have been shown to play an important role in executive labor markets that may also satisfy these criteria. CEO pensions are a significant component of overall compensation, especially when retirement is near (Bebchuk and Jackson (2005), Sundaram and Yermack (2007)), and the implicit compensation associated with investment in human capital has been shown to be an important consideration, especially in the early stages of a career (Gibbons and Murphy (1992)). We find that proxies for pension benefits are significant predictors of CDS rates for managers aged 60 years and older, while proxies for implicit compensation are significant for managers aged 59 years and younger.

With the predictions of our structural model of leverage choice in mind, we also empirically explore the relationship between CEO pay structure and leverage ratios in our sample. Coles, Daniel, and Naveen (2006) and Lewellen (2006) address the linkage from executive compensation to leverage but without the guidance of a structural model of manager choice. Convexity of compensation with respect to firm value plays a key role in their papers, and intuitive but ad hoc arguments are used to support their hypothesis that leverage should be increasing in the sensitivity of managerial wealth to stock volatility (vega). These papers find a positive relationship between vega and leverage, but a negative relationship between pay-performance sensitivity and leverage. In the context of our leverage choice model, however, these regressions would be misspecified. We instead regress leverage ratios on cash-to-stock and standard controls and find that cash-to-stock is a strong positive predictor of leverage ratios in our data. When we further allow for convexity of leverage ratios in cash-to-stock, a feature present in the parameterized version of our leverage choice model, we find statistical support but observe that the vast majority compensation terms are associated with the domain of the function where leverage ratios are increasing.

Our theoretical model builds on prior structural models of credit spreads (e.g., Merton (1974)) and of debt choice (e.g., Fisher, Heinkel and Zechner (1989)).⁷ We follow Morellec (2004) by explicitly modeling a manager-shareholder conflict, but our managerial objective function and choice set is different.⁸ The dynamic risk choice of managers in our model is closely related to the portfolio choice decision of fund managers in Carpenter (2000). Unlike in that paper, when our managers choose the initial leverage level they endogenously alter the terms of their contract. Cadenillas, Cvitanic, and Zapatero (2004) also allow a risk-averse manager to make a dynamic risk choice. Their manager is only paid with stock and shareholders, rather than managers, make the debt choice. Their model has no implications

⁷More recent structural models include Leland (1994), Leland and Toft (1996), Goldstein, Ju and Leland (2001), Collin-Dufresne and Goldstein (2001) and Titman and Tsyplakov (2003). Managers in these models are assumed to maximize value, either of the firm or of the firm's shareholders, and their decisions are typically restricted to choosing the debt amount, when to refinance, and when to default. Mauer and Triantis (1994), Leland (1998), Hennessy and Whited (2005), and Lambrecht and Myers (2006) allow managers to also make operating decisions and therefore incorporate elements of both the trade-off theory of capital structure (Modigliani and Miller (1963)) and of the agency costs of debt (Jensen and Meckling (1976)) in a dynamic setting.

⁸Several papers examine the quantitative impact of agency costs resulting from stockholder-bondholder conflicts, including Mello and Parsons (1992), Leland (1998), Parrino and Weisbach (1999), Ericsson (2000), Moyen (2000) and Hennessy (2004).

for credit spreads, since in their setting debt is endogenously risk free.

Our empirical work contributes to the extensive literature on the determinants of credit spreads (e.g., Campbell and Taksler (2003), Collin-Dufresne, Goldstein, and Martin (2001), and Elton, Gruber, Agrawal, and Mann (2001)). Our focus on CEO cash-to-stock is new in the credit spread literature, but relates to recent empirical studies on the impact of CEO pay structure, rather than CEO pay levels. Mehran (1992), Guay (1999), Coles, Daniel, and Naveen (2006), and Lewellen (2006) show that alternative measures of managerial compensation structure impact risk taking, project selection, and debt choice. Following Coles, Daniel, and Naveen (2006), Daniel, Martin, and Naveen (2005) find that credit spreads are high for firms in which CEO pay is more sensitive to stock volatility.

The remainder of the paper is structured as follows. Section 2 introduces the model. Section 3 develops closed-form expressions for the asset substitution behavior and security values. Section 4 characterizes the optimal leverage choice and links this choice to the manager's compensation terms. Section 5 examines several regressions that relate CDS rates to leverage and compensation terms. Section 6 concludes. Proofs of all propositions and other technical details can be found in the Appendix.

2 The Model

This section describes our assumptions and formally specifies the manager's optimization problems.

2.1 Market and Firm Value Dynamics

Our model is developed in a partial equilibrium, complete markets setting. We specify a pricing kernel with dynamics

$$\frac{d\xi_t}{\xi_t} = -rdt - \alpha dz_t \tag{1}$$

where r > 0 is the instantaneous risk-free rate, $\alpha > 0$ is the market price of risk, z is a standard Brownian motion, and where $\xi_0 = 1$.

We consider a firm endowed with assets whose finite life is T. The firm's manager operates this asset to generate a liquidating pre-tax payment (LPP) of V_T , representing the resale value of the asset, earnings before interest and taxes accumulated up to time T, or their sum. The LPP is shared among the firm's equity and debt claimants and also funds the payment of taxes and bankruptcy costs. We assume that the firm operates in a legal environment with limited liability for debt and equity claimants, so that $V_T \geq 0$.

The firm's manager generates the LPP by continuously controlling the instantaneous volatility of a diffusion process. We assume that all risks in the model are systematic and that the manager's actions are fully observable. As a result, the present value of the LPP will evolve according to

$$\frac{dV_t}{V_t} = (r + \alpha \nu_t)dt + \nu_t dz_t \tag{2}$$

where ν_t is the manager's time-t choice of firm risk and $V_0 > 0$ is the initial value. The controlled process ν must be based on the realized path of the unique source of risk z and, to ensure the process V_t is well defined, satisfy an integrability condition (see, for example, Karatzas, Lehoczky, and Shreve (1987)). The restriction $V_T \ge 0$ further limits the set of admissible policies ν_t . For example, $\nu_t = 1/V_t$ is not a valid choice since it gives rise to an Ornstein-Uhlenbeck process for V_t which can generate negative terminal LPP.

2.2 Taxes and Bankruptcy Costs

Taxes are assumed to be paid only at the corporate level and only at date T. To focus our attention exclusively on the tax effect of leverage, we suppose that the firm's assets are fully depreciated and that their acquisition cost has no further tax consequences. Thus in the absence of leverage taxes are proportional to LPP, i.e. $T = \tau V_T$ where $\tau \in [0, 1)$ is the firm's tax rate.⁹ If debt is issued, the taxable amount (LPP) can be reduced by the interest paid. We assume the firm's managers set debt levels by issuing a zero-coupon claim with face value L and price B_0 at the initial date. The proceeds from the issue are used exclusively to redeem outstanding shares, so that the only motivation for debt is to generate a tax shield.

Tax shields are assumed to accrue at maturity and only in states where the firm is solvent. The firm is considered solvent if bondholders and tax obligations can be paid in full, in which case the tax amount is

$$\mathcal{T} = \tau (V_T - (L - B_0)).$$

The expression $L - B_0$ represents interest paid to bondholders and leads to a tax shield of $\tau(L - B_0)$. The tax shields are thus income based, as in Kim (1978), and are consistent with the US tax code which does not allow deduction of debt principal. In case of insolvency, taxes are

$$\mathcal{T}=\tau V_T,$$

again consistent with Kim (1978) who specifies that the tax claim is senior to the debt claim and that partial payments of interest do not reduce taxable income when the firm is bankrupt. This assumption is also consistent with the "principal-first doctrine" as is mandated by the US tax code (see, e.g., Talmor, Haugen, and Barnea (1985) and Zechner and Swoboda (1986)).

Having specified the tax payments, we can define the bankruptcy threshold, V_b , by the equality $V_b = L + \tau (V_b - (L - B_0))$ which states that the LPP is just sufficient to cover

⁹It is possible to incorporate the effect of non-debt tax shields in our framework. For example, if V_0 is the undepreciated purchase cost of the asset that produces the LPP, then it would be natural to assume that the tax bill of an unlevered firm is $\mathcal{T} = \tau (V_T - V_0)$. The modified analysis would produce closed-form solutions that are qualitatively similar to those we report, but this framework would make it more difficult to isolate the pure impact of debt and its associated tax shields.

payments to bondholders and the tax authority. Rearranging the equation yields

$$V_b = L + \frac{\tau}{1 - \tau} B_0. \tag{3}$$

Solvent states are defined by $V_T \ge V_b$ and insolvent states by $V_T < V_b$. Note that when tax rates are non-zero the bankruptcy threshold is strictly above the debt face value.

We adopt the common assumption that insolvency has a direct negative impact on terminal cashflows due to a proportional deadweight bankruptcy cost $\delta_f(1-\tau)V_T$ (see, e.g., Leland (1994)).

To summarize this subsection we express the terminal state-contingent cashflows of the firm net of taxes and bankruptcy costs as

$$C_T = \begin{cases} (1-\tau)V_T + \tau(L-B_0) & \text{if } V_T \ge V_b, \\ (1-\delta_f)(1-\tau)V_T & \text{otherwise.} \end{cases}$$
(4)

This quantity is commonly referred to as free cashflow and represents funds available to pay the firm's debt and equity claimants. Under our assumptions, free cashflow is an increasing function of V_T , and it is discontinuous at $V_T = V_b$ due to bankruptcy costs and the loss of the interest tax shield in insolvent states.

2.3 Stock and Bond Pricing

We seek to derive values at any date t prior to T for the firm's terminal cashflows, equity claims and debt claims. These claims are all non-linear functions of V_T and we price them using the pricing kernel ξ . In this subsection, we take as given an arbitrary V_T satisfying the condition $V_0 = E(\xi_T V_T)$ where E is the expectation at t = 0.

We first value the firm's debt. A circularity arises because the bankruptcy threshold, V_b , depends on the initial debt value, B_0 , and the initial debt value depends on the bankruptcy threshold. To formalize this joint dependency we begin by expressing the payoff to bondholders at maturity assuming V_b is known:

$$B_T = \begin{cases} L & \text{if } V_T \ge V_b, \\ (1 - \delta_f)(1 - \tau)V_T & \text{otherwise.} \end{cases}$$

This follows from the definition of solvency in equation (3). When V_T is above V_b funds sufficient to fully repay the face value of debt are available. On the other hand, if V_T is below V_b funds are insufficient to repay the face value of debt since $(1 - \delta_f)(1 - \tau)V_T \leq$ $(1 - \delta_f)(1 - \tau)V_b$ and from equation (3) and the observation that $B_0 \leq L$, we have $(1 - \delta_f)(1 - \tau)V_b \leq (1 - \delta_f)L < L$. As a result, in the bankruptcy states bondholders recover only the cashflows available after taxes and bankruptcy costs have been incurred. The time t bond price $B_t = E_t \left(\frac{\xi_T}{\xi_t} B_T\right)$ can then be expressed as

$$B_{t} = E_{t} \left[\frac{\xi_{T}}{\xi_{t}} L \mathbf{1}_{V_{T} \ge V_{b}} + (1 - \delta_{f})(1 - \tau) \frac{\xi_{T}}{\xi_{t}} V_{T} \mathbf{1}_{V_{T} < V_{b}} \right]$$
(5)

where the notation $\mathbf{1}_{\mathbb{S}}$ is an indicator function for the event \mathbb{S} and E_t is the date-*t* conditional expectation. The bankruptcy threshold is determined at time t = 0 when the debt choice is made. Consistency with the initial bond price requires that V_b solve the non-linear equation (3) which can be restated as

$$V_b = L + \frac{\tau}{1 - \tau} E \left[\xi_T L \mathbf{1}_{V_T \ge V_b} + (1 - \delta_f) (1 - \tau) \xi_T V_T \mathbf{1}_{V_T < V_b} \right].$$
(6)

It can be shown that this condition identifies a unique $V_b \in \left[L, \frac{L}{(1-\tau)(1-\delta_f)}\right]$ given any non-negative LPP, V_T , and promised debt payment, L.

Another consequence of the definition of the bankruptcy threshold is that equity claimants receive payment only in the solvent states. To verify this, substitute definition (3) of V_b into the free cashflow equation (4) to see that $C_T = L$ when $V_T = V_b$. Thus, equity value S_T at maturity is given by

$$S_T = \begin{cases} C_T - L & \text{if } V_T \ge V_b, \\ 0 & \text{otherwise.} \end{cases}$$
(7)

This payoff can be recognized as that of a call option on terminal free cashflow with strike price equal to the face value of the bond, i.e. $S_T = (C_T - L)^+$. Alternatively, by using the same equations (3) and (4) it can be shown that terminal equity value is equivalently given by

$$S_T = (1 - \tau)(V_T - V_b)^+.$$
 (8)

This equation shows that the equity payoff can be expressed as the payoff on $(1 - \tau)$ units of a call option on LPP with strike price V_b . Time-t equity value is thus given by

$$S_t = (1 - \tau) E_t \left[\frac{\xi_T}{\xi_t} (V_T - V_b)^+ \right].$$
(9)

2.4 Managerial Compensation

Managerial compensation is assumed to have two components and is given by

$$W_T = pS_T + A,$$

where $p \in [0,1]$ is the proportion of equity granted to the manager and A > 0 is the riskless cash component of pay. The terminal payment pS_T is performance sensitive, while the payment A is performance insensitive. Without changing our qualitative conclusions, performance sensitive pay such as options or bonuses could be included in the model.¹⁰ The performance insensitive cash compensation A is assumed to be external to the firm and represents the manager's minimum pay since since stockholders have limited liability.¹¹ Performance insensitive pay takes on many forms in practice, for example salary, defined benefit pension payments, and life and health insurance benefits (Smith and Watts (1982)).¹² In practice, these payments may not be fully guaranteed, but our qualitative results will remain as long as managers receive some positive portion of the promised payments in all states.

Consistent with the view that moral hazard issues preclude borrowing against one's future income, we impose the strong restriction that managers cannot undo their compensation by trading bonds or the index.¹³ This friction will play a primary role in driving the managers choices and serves to model the agency costs associated with their inability to allocate wealth without restriction. We further assume that managers are risk averse and derive utility from terminal wealth given by

$$U(W_T) = \frac{W_T^{1-\gamma}}{1-\gamma}$$

where $\gamma > 0$ is the coefficient of relative risk aversion. Although this utility function is concave in wealth, derived utility is not a globally concave function of V_T unless debt levels are zero. In fact, when debt is present and a firm is near its bankruptcy threshold the manager's derived utility is locally convex. Therefore, as in other models of risk-shifting behavior, there will be an incentive to increase LPP risk in these states. Concavity in high payoff states will mitigate this incentive, however, so, unlike in the case where the manager maximizes shareholder value, an infinite volatility choice will be suboptimal.¹⁴

¹⁰With stock options in the model terminal wealth would be given by $W_T = pS_T + q(S_T - K)^+ + A$ where q is the number of executive stock options with strike price K. This modification adds complexity and does not qualitatively affect the manager's incentives since stock of a levered firm alone makes the manager's pay convex in firm value.

¹¹Without changing the solution to the model, we could alternatively view the firm's assets as segregated into two accounts: one that the manager can control and to which he is an equity holder with proportion p, worth V_0 , and another that he cannot control, to which he is the most senior claimant, and is riskless with value $e^{-rT}A$.

¹²Many CEOs negotiate explicit employment agreements that stipulate minimum salaries and provide protection to this amount in the case of dismissal (Gillan, Hartzell, and Parrino (2008)). Firms are also able to fully guarantee executive pension assets using various legal means (Bebchuk and Fried (2004)).

¹³This restriction can be weakened if we work in a more realistic setting where the manager can undo some, but not all, of his compensation. This would hold if firm risk has an idiosyncratic component that the managers cannot hedge or if partial borrowing against their terminal payout is permitted. The economic intuition from our model will hold in such a setting but the added complexity would obscure the results.

¹⁴In a seminal paper, Green (1984) analyzes the risk choice of stockholders when both debt and convertible stock are present. The conversion feature reverses the convexity in levered equity when earnings are high which, as in our setting, mitigates the incentive to take risk. Although we utilize risk aversion to make the manager's objective concave for high values of LPP, Green (1984) shows that similar objective functions can arise in settings where managers maximize shareholder value and there are explicit or implicit options on LPP embedded in this value. We postulate that our qualitative findings are robust to such interpretations.

2.5 The Manager's Choices

We assume that self-interested managers commit to a leverage level and then alter project risk to maximize their derived utility from compensation. It is convenient to consider these choices as occurring in two distinct stages. Figure 1 summarizes the sequence of events.

In the second stage of the problem the manager has pre-committed to a fixed level of debt L, which has been sold for its fair market value B_0 , leaving him with a proportion p of the firm's equity. He now controls the volatility ν of the LPP process. This problem is formally stated as

$$J(L) \equiv \sup_{\nu} E\left[U(A + p(1 - \tau)(V_T - V_b)^+)\right],$$
(10)

where V_T is the non-negative LPP generated by (2) and where V_b is given by (6).

In the first stage of the decision problem managers announce the face value of debt (which cannot be later changed), sell it to outside investors, and repurchase outstanding equity with the proceeds. Managers cannot sell their own stock, so an increase in debt levels will be accompanied by an increase in their proportion of the firm's outstanding equity. We assume that investors have full information about the manager and his compensation, and impose rational expectations so that investors correctly forecast the second stage volatility choice. Debt and equity are thus fairly priced, and the value of the firm after the leverage announcement L is given by $C_0 = E(\xi_T C_T)$ where C_T is defined by equation (4) and V_T in that equation is the correctly anticipated outcome of the manager's second stage optimization.

We assume that the manager is endowed with an initial equity proportion in the unlevered firm, p_0 . He credibly announces his intent to issue debt with face value L which immediately causes the valuation impact of tax shields, bankruptcy costs, and agency effects of debt to be incorporated into C_0 . The manager's initial dollar holdings of equity are then equal to p_0C_0 . To identify the manager's proportional holdings of the levered firm, $p > p_0$, notice that debt proceeds of B_0 replace outside equity worth the same amount. After recapitalizing, the total value of the firm's equity decreases to $S_0 = C_0 - B_0$ and the value of the manager's stock-holdings can be expressed as $p(C_0 - B_0)$. This logic defines pthrough the equation

$$p_0 C_0 = p(C_0 - B_0). (11)$$

The manager's first stage problem can now be defined formally as

$$\sup_{L} J(L) \tag{12}$$

subject to the non-linear constraint (11) relating p and L. Two opposing forces are at play in this problem. Risk averse managers wish to trade-off risk and return so as to optimize their terminal wealth. Since they are precluded from undoing their compensation, managers alter their wealth by controlling initial leverage and firm volatility. Increasing leverage provides a tax benefit that directly increases firm value and, because managers hold a proportion p_0 of the firm, this increases their wealth. Compensation is convex, however, and will provide an incentive to increase risk in certain states. Risk averse managers anticipate these actions and so they have an incentive to choose low leverage in the first stage. These trade-offs give rise to predictions relating the manager's compensation to leverage choice, risk choice and credit spreads that we explore in the next sections.

3 The Effects of Managerial Compensation and Asset Substitution on Security Valuation

In this section, we take as given a fixed level of debt and analyze the impact of the manager's compensation package on his risk choice in the second stage optimization problem (10). Our problem admits a closed-form solution, and we explicitly demonstrate how managers engage in asset substitution by choosing instantaneous firm risks contingent on current firm value. This action affects all asset values and we focus our analysis on the implications for credit spread dynamics.

3.1 Characterizing Asset Substitution

The manager has no ability to alter the present value V_0 of the LPP but has complete freedom to allocate the LPP across states summarized by ξ_T . He accomplishes this task in a complete market by continuously controlling the instantaneous volatility ν . This mirrors the technique by which derivative payoffs are synthesized by changing the exposure of the replicating portfolio to the underlying assets.

Following Karatzas, Lehoczky, and Shreve (1987) and Cox and Huang (1989), the manager's dynamic second stage problem (10) is equivalent to the static choice problem

$$\sup_{V_T} E\left[\frac{(A+p(1-\tau)(V_T-V_b)^+)^{1-\gamma}}{1-\gamma}\right],$$
(13)

subject to the budget constraint $E(\xi_T V_T) \leq V_0$, the V_b definition in equation (6), and the non-negativity constraint $V_T \geq 0.^{15}$ This is the risk shifting (R) model and we denote its solution by V_T^R . This solution will generate a bankruptcy threshold V_b^R as well as LPP (V_t^R) , equity (S_t^R) , and bond (B_t^R) processes.

If V_b in problem (13) is an exogenous constant then the presence of the kink in utility derived from V_T at V_b induces a local convexity in the objective function. Carpenter (2000) shows that the optimal choice under this restriction can be obtained by maximizing the

¹⁵This formulation of the second stage problem clarifies that if no lower bound on V_T is imposed, the manager can short-sell an arbitrarily high amount of LPP in one state to finance the acquisition of a large positive amount of LPP in another state. This will increase utility, since the manager's derived utility will be $A^{1-\gamma}/(1-\gamma)$ in the negative LPP state and can be made arbitrarily large in the other state. The maximization problem (13) is then not well defined. Although we can solve for the optimal choice given any constraint $V_T \geq \underline{V}$ for any constant \underline{V} , we focus on $\underline{V} = 0$ which has the natural interpretation of a limited liability constraint.

concavified objective function, defined as the smallest concave function dominating the utility derived from compensation in problem (13) (see Aumann and Perles (1965)). The bankruptcy threshold V_b is not exogenous in our setting, however, since its value depends on the manager's choice V_T . This circularity between problem (13) and equation (6) is not present in the Carpenter (2000) model and is addressed by the following proposition.

Proposition 1 Given any $(L, p) \in (0, \infty) \times [0, 1]$, there exists a unique rational expectations equilibrium in which bondholders correctly anticipate the manager's risk choice ν . In this equilibrium the bankruptcy threshold V_b^R is consistent with (3) and the optimization problem (13) yields firm LPP

$$V_T^R = \left[\overline{V} + \left(\overline{V} - V_b^R + \frac{A}{p(1-\tau)}\right) \left(\left(\frac{\xi_T}{\overline{\xi}}\right)^{-\frac{1}{\gamma}} - 1\right)\right] \mathbf{1}_{\{\xi_T \le \overline{\xi}\}},\tag{14}$$

where $\overline{V} \in (V_b^R, \infty)$ is the unique solution to the nonlinear equation

$$\left[\frac{[A+p(1-\tau)(\overline{V}-V_b^R)]^{1-\gamma}}{1-\gamma} - \frac{A^{1-\gamma}}{1-\gamma}\right]\overline{V}^{-1}$$
$$= p(1-\tau)\left[A+p(1-\tau)(\overline{V}-V_b^R)\right]^{-\gamma},$$
(15)

and where $\overline{\xi} \in (0,\infty)$ is the unique scalar for which the optimal LPP defined by equation (14) satisfies the budget constraint $E\left(\xi_T V_T^R\right) = V_0$.

This proposition characterizes the manager's second-stage behavior. The quantity \overline{V} has a natural geometric interpretation. As shown in the proof of Proposition 1, the concavified objective is linear and greater than the derived utility for $V_T < \overline{V}$ and coincides with the derived utility function for $V_T \geq \overline{V}$. The requirement that the concavified utility function is the smallest concave function dominating the derived utility imposes equation (15) which ensures tangency of the two functions at \overline{V} . Managers therefore behave as if they are risk neutral over terminal payoffs in the range $[0, \overline{V}]$ and, as a result, any LPP lying within this region is dominated by an LPP with the same present value but divided between a zero payoff state and states with payoffs larger than \overline{V} . This implies that the optimal LPP will never lie in the interval $(0, \overline{V})$ since, by equation (14), V_T^R is decreasing in ξ_T , equal to \overline{V} at $\xi_T = \overline{\xi}$, and zero for $\xi_T > \overline{\xi}$. This also shows that the manager sells payments from states where the pricing kernel is larger than the critical value $\overline{\xi}$ and uses the proceeds to buy payments in states where the pricing kernel is lower than $\overline{\xi}$. The probability of a zero payoff is, therefore, strictly positive and equals the probability of the event $\xi_T > \overline{\xi}$.

Proportional bankruptcy costs δ_f do not play a role in Proposition 1 since the manager's optimal behavior leads to zero recovery for bondholders in bankruptcy. This follows from the fact that the smooth pasting condition in equation (15) yields a solution \overline{V} that is above the bankruptcy threshold V_b^R . The counterfactual absence of bondholder recovery in bankruptcy is due to the assumption that the manager can perfectly control the instantaneous volatility

 ν_t of V_t . Relaxing this assumption, for example by limiting the possible choices of ν_t , will give rise to partial recovery for bondholders but the qualitative nature of the optimal LPP in Proposition 1 will remain.¹⁶ A robust economic consequence of risk shifting is that value is transferred from bankrupt into solvent states, so despite the fact that the exact form of the optimal LPP in equation (14) is not realistic, it highlights in a qualitative way an important aspect of this class of models. Another possibility for incorporating partial recovery in a structural way is to include in managerial compensation a bankruptcy penalty of the form $\delta_m (V_b - V_T)^+$ where δ_m is a positive constant. This generalization of the model would produce a set of insolvent states where firm value is positive and another set of insolvent states where firm value is zero. We do not further examine either of these modifications of our model in order to maintain clarity in the exposition of our main economic findings.

To provide a benchmark terminal LPP we now consider the manager's choice if given compensation comprised solely of a proportion \hat{p} in the stock of an unlevered firm, that is if the manager solves

$$\sup_{V_T} E\left[U(\hat{p}(1-\tau)V_T)\right] \tag{16}$$

subject to $V_0 = E(\xi_T V_T)$. We refer to this problem as the "Unlevered Benchmark" and, to ensure that the present values of compensation in problems (13) and (16) are the same, we set the equity proportion \hat{p} to solve $\hat{p}(1-\tau)V_0 = pS_0^R + e^{-rT}A$. The optimal state-contingent LPP can be written as $V_T^M = (\lambda\xi_T)^{-1/\gamma}$ where λ is identified by solving $V_0 = E(\xi_T V_T^M)$. The value function from the Unlevered Benchmark is at least as high as in the second stage problem (13) because it is possible to run the unlevered firm so that the manager's stake $\hat{p}(1-\tau)V_T$ exactly matches the optimal terminal compensation $A + p(1-\tau)(V_T^R - V_b^R)^+$ from the R model.

Figure 2 contrasts the manager's behavior in the second stage problem to that in the Unlevered Benchmark by plotting terminal compensation against the terminal value of the pricing kernel. The two compensation schedules cross twice, at the points $\overline{\xi}$ and $\xi^* > \overline{\xi}$ (not displayed), consistent with the fact that they have the same present value. The figure shows that since the manager optimally sets $V_T^R = 0$ when $\xi_T > \overline{\xi}$, he receives the minimal compensation A in those states. To achieve the higher utility associated with the Unlevered Benchmark solution he would like to sell claims from states in which the pricing kernel is high (i.e. when $\xi_T > \xi^*$), but this would require him to sell the fixed component of his pay which is not permitted. Figure 2 thus highlights that the important friction generating asset substitution derives from the assumption that the manager cannot borrow against the cash component of his compensation using stocks or bonds.

¹⁶We have analyzed the simple class of policies where the manager can choose between only two values of volatility $\nu_t \in \{\underline{\nu}, \overline{\nu}\}$. Low volatility $\underline{\nu}$ is chosen unless V_t is below a critical value V^* , in which case the manager sets volatility to its high level $\overline{\nu}$. This less flexible control does not allow the manager to achieve the optimal terminal LPP described in equation (14). Simulations using the restricted policies, however, show that the probabilities of terminal LPP being in a region near zero or above V^* are relatively high, and the probability of being just below V^* is low. This verifies that terminal payouts similar to the optimal LPP achieved in complete markets can be obtained even if perfect control of volatility is not possible.

We now explicitly characterize risk taking behavior in the second stage:

Proposition 2 For any time $t \in [0,T)$, the manager's optimal volatility choice is given by

$$\nu_t^R = \left(\overline{V} - V_b^R + \frac{A}{p(1-\tau)}\right) e^{-(r+\alpha^2/(2\gamma))\gamma^*(T-t)} \left(\frac{\xi_t}{\overline{\xi}}\right)^{-1/\gamma} \\
\times \left[\frac{\alpha}{\gamma}\mathcal{N}(d(t,\gamma^*,\xi_t/\overline{\xi})) + \frac{n(d(t,\gamma^*,\xi_t/\overline{\xi}))}{\sqrt{T-t}}\right] \frac{1}{V_t^R} \\
+ \left(V_b^R - \frac{A}{p(1-\tau)}\right) e^{-r(T-t)} \frac{n(d(t,1,\xi_t/\overline{\xi}))}{V_t^R\sqrt{T-t}},$$
(17)

where $\gamma^* = 1 - 1/\gamma$,

$$d(t, x, m) = \left(-\ln m + r(T - t) - \frac{\alpha^2}{2}(1 - 2(1 - x))(T - t)\right) / (\alpha\sqrt{T - t}),$$

 $\mathcal{N}(.)$ and n(.) are the standard normal cumulative distribution and density functions, and where the present value at time t of LPP is given by

$$V_t^R = \left(\overline{V} - V_b^R + \frac{A}{p(1-\tau)}\right) e^{-(r+\alpha^2/(2\gamma))\gamma^*(T-t)} \left(\frac{\xi_t}{\overline{\xi}}\right)^{-1/\gamma} \mathcal{N}(d(t,\gamma^*,\xi_t/\overline{\xi})) + \left(V_b^R - \frac{A}{p(1-\tau)}\right) e^{-r(T-t)} \mathcal{N}(d(t,1,\xi_t/\overline{\xi})).$$
(18)

Risk choice is illustrated in Figure 3, where time-t LPP volatility ν_t^R is plotted against the present value of LPP V_t^R/L for two cash-to-stock levels. In the graph, we also plot the standard deviation chosen in the Unlevered Benchmark which can be shown to be the constant $\nu^M = \alpha/\gamma$. We see that when V_t^R/L is low firm volatility is above ν^M . When stock is deep out-of-the-money, asset substitution incentives become very strong and the firm becomes extremely risky. As V_t^R/L increases, firm stock moves in-the-money and the benefit of deviating from the Unlevered Benchmark strategy falls. In the limit when stock is deep in-the-money, the manager volatility choice converges to the Unlevered Benchmark volatility. This is intuitive because in these states stock is the dominant component of compensation and leverage ratios are low. The graph also shows that volatility choice is not necessarily a monotonic function of V_t^R/L . In particular, if the ratio of cash-to-stock is relatively low we observe a U-shaped function whereby volatility can be reduced below even the Unlevered Benchmark level. This reflects the fact that compensation can actually induce conservative behavior when the manager acts to keep his firm's stock in-the-money and secure final pay above A.

3.2 Security Valuation in the R Model

To value bonds in the R model, recall that at the terminal date the firm will either be solvent with $V_T^R \ge \overline{V} > V_b^R$, or insolvent with $V_T^R = 0$. Bondholders will thus receive the face value L when the firm is solvent and zero otherwise, and equation (5) produces the bond price formula

$$B_t^R = Le^{-r(T-t)} \mathcal{N}(d(t, 1, \xi_t/\overline{\xi})) \tag{19}$$

for any $t \in [0, T]$, where the function d is defined in Proposition 2. The last term in this expression is the risk-neutral probability that the firm will be solvent and, when discounted, represents the value of an Arrow-Debreu security paying \$1 in the solvent states $\xi_T < \overline{\xi}$. This observation accounts for the particularly simple form of the pricing function, which discounts the face value payments bondholders receive in non-bankrupt states. Bond yields in the model can be immediately deduced from the bond price and are given by

$$y_t^R \equiv -\frac{\ln(B_t^R/L)}{T-t} = r - \frac{1}{T-t}\ln(\mathcal{N}(d(t, 1, \xi_t/\bar{\xi}))).$$
(20)

The corresponding credit spread is

$$\rho_t^R \equiv y_t^R - r = -\frac{1}{T-t} \ln(\mathcal{N}(d(t, 1, \xi_t/\overline{\xi}))).$$
(21)

In addition to its effect on credit spreads, asset substitution also has a direct impact on bond return volatility. Using Itô's Lemma, the instantaneous standard deviation η_t of the bond return can be shown to be

$$\eta_t^R = \frac{1}{\sqrt{T-t}} \frac{n(d(t,1,\xi_t/\overline{\xi}))}{\mathcal{N}(d(t,1,\xi_t/\overline{\xi}))}.$$
(22)

To value stock in the R model, recall that stockholders are paid zero in insolvent states and $(1 - \tau)(V_T^R - V_b^R)$ in the solvent states (see equation (8)). Proposition 1 shows that V_T^R is zero in insolvent states $\xi_T > \overline{\xi}$ and $V_T^R \ge \overline{V} > V_b^R$ in solvent states $\xi_T \le \overline{\xi}$, hence $S_T^R = (1 - \tau)(V_T^R - V_b^R)^+ = (1 - \tau)(V_T^R - V_b^R \mathbf{1}_{\xi_T \le \overline{\xi}})$. This implies that the stock payout can be replicated by a long position in $(1 - \tau)$ units of a claim to V_T^R and a short position in $(1 - \tau)V_b^R/L$ units of the firm's bond. Assuming no arbritage, equity price in the R model must be given by

$$S_t^R = (1 - \tau) \left(V_t^R - V_b^R \frac{B_t^R}{L} \right).$$
(23)

Application of Itô's Lemma to this equation identifies the volatility of stock returns, given by

$$\sigma_t^R = \frac{\nu_t^R V_t^R - \eta_t^R V_b^R B_t^R / L}{V_t^R - V_b^R B_t^R / L}.$$
(24)

3.3 Cross-sectional Implications of the Second Stage Problem

We now describe how, in a cross-section of firms, the R model relates leverage ratio $L/(S_0^R + L)$ and the model parameters $(A, p, L, V_0, \alpha, \gamma, \tau, r, T)$ to endogenously determined credit spreads, stock return volatilities, and corporate debt hedge ratios.

We begin with the following proposition.

Proposition 3 For any admissible parametrization $(\alpha, \gamma, \tau, r, T)$, credit spreads in the R model may be expressed as

$$\rho_0^R = f\left(\frac{A}{pS_0^R}, \frac{L}{S_0^R + L}, \alpha, \gamma, \tau, r, T\right)$$
(25)

where f is a increasing function of both cash-to-stock $A/(pS_0^R)$ and leverage ratio $L/(S_0^R+L)$.

This proposition shows that the relevant incentive for risk shifting in the R model is measured by the level of cash compensation A relative to the managerial stockholdings pS_0^R . As a result, the impact of the manager's compensation parameters (A, p) on credit spreads can be summarized by the single measure cash-to-stock $A/(pS_0^R)$.¹⁷ The proposition also shows that it is not necessary to directly observe the present value of the LPP process V_0 in order to determine credit spreads. Proposition 3, therefore, makes the clear empirical prediction that in a cross-section of firms credit spreads should be positively related to cash-to-stock when controlling for leverage ratio and other characteristics of the firm and its manager. This prediction forms the main hypothesis in the empirical analysis of Section 5.

In order to understand the economic mechanism driving Proposition 3 we undertake a simple comparative static exercise. We consider firms that conform to the R model and have in common the exogenous parameters $(\alpha, \gamma, \tau, r, T)$. We then analyze two subsamples, the first with a common high level of cash-to-stock and the second with a common low level of cash-to-stock. For each subsample, we vary the face value of of debt L and the initial LPP V_0 and, by utilizing the valuation formulas of Section 3.2, numerically depict equation (25) which relates leverage ratios and credit spreads.

Parameters are calibrated as follows: We consider debt with an initial maturity of T = 5 years. The riskless rate r = 5% p.a. is comparable to its recent level in US markets. The market price of risk is set to $\alpha = 0.33$, consistent with a risk premium of 7% p.a. and market volatility of 21% p.a. The tax rate is set to $\tau = 30\%$ which approximates the marginal statutory rate for a fully taxable US corporation. The cash-to-stock ratios are $A/(pS_0^R) \in \{0.02, 0.10\}$, which are the median and 90th percentile levels in our sample of US-based firms (see Section 5). The risk aversion parameter $\gamma = 1.1$ is chosen so that for a firm run by a manager with the median cash-to-stock $A/(pS_0^R) = 0.02$ and with a leverage ratio equal to the median value of $L/(S_0^R + L) = 26.8\%$, the stock volatility approximately equals the sample median value of $\sigma_0^R = 30\%$. A manager with $\gamma = 1.1$ is slightly more risk averse than a manager having logarithmic utility.

Panel A of Figure 4 presents the relationship between credit spreads and leverage ratios for this cross-section of firms and Panel B shows how leverage ratios of the firms relate to

¹⁷In a cross-section of firms, a manager with a fixed cash payment A and a fractional claim p to a large firm will behave differently from one with the same A and p but employed at a small firm. The proof of Proposition 3 shows that the scaling $A/(pS_0^R)$ appropriately accounts for the effect of firm size on manager behavior. In theory, other scalings like A/(pL) or $A/(pV_0)$ can also provide sufficient statistics for compensation terms and a result similar to Proposition 3 also holds when these ratios replace the ratio $A/(pS_0^R)$. We choose stock price as a scaling parameter because it is consistent with our interpretation of $A/(pS_0^R)$ as a cash-to-stock ratio.

equity volatility. The figure shows that given any leverage ratio, increasing cash-to-stock elevates credit spreads. To understand this result, consider a firm with median cash-to-stock $A/(pS_0^R) = 0.02$, a given leverage ratio $L/(S_0^R + L)$, and the corresponding credit spread ρ_0^R . Assume that cash-to-stock for this manager increases to $A/(pS_0^R) = 0.10$, for example by increasing cash compensation from A to 5A while holding constant p and S_0^R . If the LPP V_0 is held constant, the immediate impact of this change in A is to increase the manager's risk shifting incentive, consistent with the intuition that greater insurance against bad outcomes reduces effective risk aversion. This behavior transfers value from bondholders to stockholders. Bond prices would thus decrease, causing credit spreads to increase. Stock prices would also increase, so if we wish to consider the impact of increasing cash-to-stock on credit spreads holding S_0^R constant a reduction in V_0 is necessary. This will further increase credit spreads, because holding L constant and reducing the time-0 value of the LPP results in higher bankruptcy probabilities and higher spreads.

Panel A of Figure 4 also illustrates that for each level of cash-to-stock, increasing the leverage ratio in the R model has a positive impact on credit spreads, as one would expect. In addition, the figure shows that spreads generated by the R model can be substantially higher than those from a model where the manager does not risk shift and LPP volatility is constant. In the appendix, we formally develop an alternative "M model", which is a version of the Merton (1974) model incorporating taxes and bankruptcy costs. We set the proportional bankruptcy cost to its maximal value $\delta_f = 100\%$ to generate an upper bond for the M model's prediction of credit spreads. Each firm in the R model subsample with median $A/(pS_0^R) = 0.02$ is matched to an M model firm with identical equity volatility $\sigma_0^M = \sigma_0^R$. This is achieved by appropriately choosing the M model's LPP volatility. The curve labeled "M model" in Panel A of Figure 4 plots credit spreads for this set of matching firms. The figure shows that, at all leverage levels, spreads in the R model with median cash-to-stock are larger than spreads in the M model. The R model can, therefore, be parameterized to address the main shortcoming of traditional structural models pointed out by Eom, Helwege, and Huang (2004) namely that predicted spreads are too low relative to observed spreads. Note that the R model does not mechanically increase spreads by simply elevating LPP and stock risk. In fact, equity volatility for each matched pair from the R and M models are by construction identical and yet spreads are higher in the R model. For example when leverage is 50%, the R model's manager is running the firm so that equity volatility is as in the M model (about 30%) but spreads are 50 bp larger in anticipation of future risk shifting that transfers value from bondholders to stockholders. ¹⁸

¹⁸The basic economic mechanism that inflates credit spreads in the R model is that the manager can undertake non-contractible activity that decreases bondholder value. In the model of Collin-Dufresne and Goldstein (2001), managers in a Merton (1974) setting have the option to issue *pari-pasu* debt at an intermediate date. Bondholders realize that this action may increase the probability of bankruptcy and dilute their payments in insolvent states. They therefore pay less for debt issued by a firm that can recapitalize in the future than for the debt of a firm that is restricted from issuing an equal or senior claim. An interesting extension of our model would allow managers to optimally recapitalize at an intermediate date. Our intuition is that this activity would serve to further inflate credit spreads in the R model.

Structural models provide practical guidance for hedging debt and, perhaps more importantly, for hedging credit-sensitive derivative securities like credit default swaps. The hedge ratio for debt relative to equity is given by the closed form formula $\frac{\eta_0^R B_0^R}{\sigma_0^R S_0^R}$. Figure 5 compares bond/stock hedge ratios for our two parameterizations of the R model and for two parameterizations of the M model. The figure shows that the models produce very different hedge ratio. Cash-to-stock therefore is an important determinant of hedge ratios and failing to incorporate information on compensation terms can result in poor quality hedging. In addition, the figure also highlights that hedge ratios can provide additional informative empirical moments for tests of structural models.¹⁹

4 The Optimal Leverage Choice

In this section, we analyze the manager's leverage choice in the first stage optimization problem (12), imposing rational expectations. As in Leland (1998), leverage is chosen with correct anticipation of future operating decisions, and securities are fairly priced beginning from the instant the leverage commitment is made.

We begin our study of the manager's optimal leverage choice by analyzing the objective function after the manager has pre-committed to an arbitrary debt face value, L. Figure 1 describes the sequence of events that follows the commitment. Debt is sold at its fair market value and the proceeds are used to repurchase outside stock, a refinancing activity that increases the manager's equity share p as given by equation (11) and ultimately alters his second stage behavior. The manager's choice of the LPP risk process ν_t^R thus depends on debt and equity prices, but since debt and equity prices depend on the managers *ex-post* LPP choice we have a feedback loop in our model. We formally address this interdependency in the following proposition which establishes the existence of a unique rational expectations equilibrium.

Proposition 4 Fix a leverage level L > 0 and assume that

$$p_0 < 1 - \left(\frac{B_0^R}{C_0^R}\right)_{p=1} \tag{26}$$

where the term $\left(\frac{B_0^R}{C_0^R}\right)_{p=1}$ is the ratio of bond prices to the firm free cashflow value with no outside investors. Then there exists a unique rational expectations equilibrium in which bondholders and outside equity holders correctly anticipate both the manager's risk choice ν^R and the final equity proportion $p \in [p_0, 1]$ that is consistent with equation (11). In this equation $B_0 = B_0^R$ as specified in equation (19) and $C_0 = B_0^R + S_0^R$ where S_0^R is given by equation (23).

The inequality (26) in the proposition rules out cases where the debt issuance completely

 $^{^{19}}$ Schaefer and Strebulaev (2004) compare observed to predicted hedge ratios for bonds and find support in the data for the Merton (1974) model.

eliminates the holdings of outside shares. For instance, this condition is satisfied when p_0 is small or T is large.

Besides establishing existence of the equilibrium, the proof of the proposition provides guidance for constructing a numerical procedure to solve the first-stage optimal capital structure problem. Given any candidate debt level L, a corresponding manager equity share p can be found using an iterative process. The derived utility of compensation can then be calculated. Hence, a simple univariate unconstrained optimization routine can be employed to determine the optimal debt choice.

Managers trade off two opposing effects when selecting firm leverage. On one hand, their wealth is increasing in leverage due to the tax shield. Managers are risk averse, however, and they internalize the fact that their leverage decision will pre-commit them to risk shift, thereby penalizing their derived utility.²⁰ The purpose of our numerical exercise is to quantify which of the two forces is dominant under various compensation terms.

The top panel in Figure 6 depicts the relationship between optimal leverage and the cashto-stock ratio of the manager's compensation, $A/(pS_0^R)$.²¹ This relationship is U-shaped. The cash component of compensation provides the manager with insurance, so when A/pS_0^R is high the manager's effective risk aversion is relatively low. In such cases he perceives the cost of increasing the face value of debt to be low. Optimal leverage is, therefore, increasing in A/pS_0^R in this region. The bottom panel in Figure 6 relates credit spreads for newly issued debt to A/pS_0^R . This function is upward sloping and the magnitude of credit spreads are large when A/pS_0^R is large. This implies that managers with compensation that is relatively safe are issuing very risky debt.

An interesting implication of Figure 6 is that when the cash component of compensation is low, managers choose to issue high levels of safe debt. This behavior can be understood by considering the special case in which $A/pS_0^R = 0$. Managers will avoid running their firm into bankruptcy at all costs, since their marginal utility is infinite at zero wealth. It seems counterintuitive that they would then add leverage. This intuition does not account, however, for the fact that they can avoid bankruptcy by controlling volatility to ensure firm value exceeds a fixed lower bound at the terminal date. Following this logic, when A/pS_0^R is low, the manager's second stage choice of LPP results in a firm with low risk relative to the case where A/pS_0^R is high (see Figure 3). This behavior has the effect of weakening the risk-based disincentives of debt. As a result, these firms will have relatively high leverage ratios and, simultaneously, low asset risk. Such endogenous substitution between leverage and risk choice is to be expected in any model with risk-averse managers. The complete

 $^{^{20}}$ Bankruptcy costs do not affect optimal leverage choice because risk-shifting causes firm value to be zero in insolvent states. A more general version of the model that incorporates a bankruptcy penalty for the manager would give rise to partial recovery, in which case optimal debt would trade off tax shields with bankruptcy costs and risk aversion.

 $^{^{21}}$ The technical appendix shows that the first stage model exhibits an homogeneity property supporting the approach of Figure 6 which represents leverage ratio and credit spreads as functions of cash-to-stock after the issuance of debt. The figure is qualitatively unchanged when values of the fixed parameters are changed within a reasonable range.

market assumption in the R model, which is analogous to allowing unrestricted choice of LPP volatility, makes this behavior stark in our model, but the underlying economic forces will also be present in models where this assumption is relaxed.²²

Panel A of Figure 6 also illustrates that optimal leverage ratios vary dramatically when risk shifting incentives change. This contrasts with the analysis in Leland (1998), where the impact of asset substitution on optimal leverage is low. Another important empirical implication of the model is that low optimal leverage ratios can be produced. With a parametrization similar to Leland (1998) our managers can optimally choose leverage ratios on the order of 20% or less, while in Leland optimal leverage ratios are more typically in the 50% range. Panel B shows that at these low leverage levels, credit spreads can vary significantly. For example, we see that if a manager chooses to issue debt with a face value around 0.2, implying a debt-to-value ratio for the levered firm on the order of 20%, the credit spread can be as low as 2% or as high as 15%, depending on the cash-stock mix in compensation. This highlights that compensation terms interact with the choice of leverage and asset substitution, so that the riskiness of debt is not only determined by firm characteristics but also by observable aspects of the manager's pay.

5 Does Managerial Compensation Influence CDS Rates?

Figure 7 provides a visual summary of firm-year average CDS rates as a function of prior year-end leverage ratios for our sample. As predicted by typical structural models (e.g. Merton (1974)), CDS rates are generally increasing in the leverage ratio proxy. The figure also shows that significant cross-sectional variation in CDS rates remains even after controlling for leverage. Our empirical analysis addresses whether cross-sectional variation in risk shift-ing motives, as made precise in the R model, can provide a plausible economic mechanism to explain this additional variability.

The main result in this section is the finding of a robust positive association between cash-to-stock and CDS rates in a sample of US-based firms. This new empirical finding provides direct support for the R model prediction illustrated in Panel A of Figure 4 and formalized in Proposition 3.

²²For example, we have considered a setting in which the manager chooses debt levels in the first stage and makes a one-time second stage decision regarding firm risk. The risk choice is limited to be one of two possible values ν_l or $\nu_h > \nu_l$. Managers cannot avoid bankruptcy using this limited set of policies since the resulting LPP is log-normal, so low debt values are chosen when A/pS_0^R is very small. In an unreported numerical exercise we verify, however, that managers with low (high) levels of A/pS_0^R issue relatively large amounts of debt and choose low (high) LPP volatility, resulting in low (high) credit spreads. At moderate levels of A/pS_0^R low debt levels are chosen. The U-shape in Figure 6 is, therefore, not exactly replicated, but the qualitative cross-sectional predictions remain unchanged.

5.1 Data

To construct our sample, we begin with a database of daily CDS rates collected by the Markit Group during the period January 2, 2001, to December 21, 2006.²³ We follow the common practice of using the five-year maturity contract whose liquidity is high relative to other maturities (see, for example, Berndt *et al.* (2005)). In addition, we restrict our attention to senior unsecured debt of US-based issuers to minimize legal and operating environment heterogeneity. The database includes companies that default during this time period (e.g., Enron), thus mitigating the effect of survivorship bias. Using this data, we calculate a CDS rate for each firm-year by averaging the end-of-month observations.

Accounting, compensation, and financial data is obtained by manually linking the Markit database to the merged CRSP/Compustat database using company names. All firms in our sample are required to have outstanding publicly-traded ordinary common shares (CRSP) share code 10 or 11). Computat's ExecuComp database provides information for each CEOcompany-year combination on salary, equity holdings, option holdings, and total shares outstanding. We then divide equity and option holdings by total shares outstanding to create proportional ownership of stock p_s and options q. To create a single measure of a manager's effective stock ownership we calculate the proxy $p = p_s + \Delta q$, where Δ is the hedge ratio of the manager's option holdings, calculated using the procedures of Core and Guay (2002). The method of Core and Guay (2002) is also used to calculate average option exercise prices K for each CEO, from which we calculate the moneyness K/S using end-ofyear stock prices. ExecuComp provides information for most of the CEOs in our dataset on age, hiring date, and the date when the title CEO first applies. In cases where any of these three variables were missing, the SEC-edgar or ZoomInfo.com websites were searched using the CEO and company names from ExecuComp. CEOs are considered newly hired (New CEO = 1) if they are within one calendar year of their appointment date. Debt proxies for each sample firm are defined by the sum of debt in current liabilities and total longterm debt (Compustat industrial annual data items 9+34). This debt proxy is converted to a market leverage ratio (L/(S+L)) using calendar year-end CRSP market capitalization values. Sales, return on assets (ROA), and equity returns r for each fiscal year are taken directly from ExecuComp. Collateral is calculated as the ratio of inventory plus PP&E to assets (Compute data items (3+8)/6). Book-to-market B/M is calculated using the procedure outlined in Daniel and Titman (1997). To measure the current level of stock volatility we calculate the standard deviation of daily stock returns each year at the firm level using CRSP returns from the fourth quarter. Also at the firm level, we calculate the ratio of average monthly CRSP excess returns to their standard deviation during each calendar year. This proxy for the firm's Sharpe ratio SR is, in principle, time invariant. To improve its precision we calculate for each firm-year the lagged moving five-year average. The Gompers, Ishii, and Metrick (2003) "G-index" is obtained from the website of Andrew

²³Other studies that utilize the Markit database include Berndt, Lookman and Obreja (2006), Duarte, Longstaff and Yu (2007), and Yu (2006).

Metrick and linked to our database using ticker symbols. The index G is an indicator of the balance of power in a takeover event between shareholders and managers ranging from zero for shareholder-friendly firms to 24 for manager-friendly firms.²⁴ Simulated marginal corporate tax rates before financing are obtained from the website of John Graham.²⁵ Missing simulated tax rates are filled in using the procedure of Graham and Mills (2008). Our resulting base sample consists of 608 firms.

Panel A of Table I summarizes our raw data on the cross-section of firm characteristics. Firm average CDS rates have a mean of 153 bp and a median of 70 bp, from which we infer that the distribution is right skewed. Average market leverage, sales, market capitalization, book-to-market and return-on-assets are close to comparable figures for S&P 500 firms during the same period.

Panel B of Table I summarizes the CEO data. Average salary, stock holdings excluding stock held indirectly through options (p_s) , and shares underlying option holdings (q) are approximately equal to the averages for S&P 500 firms. Effective stock ownership p has a mean that is above that of p_s and below that of $p_s + q$ as would be expected given its definition as $p = p_s + \Delta q$. Average CEO age, tenure, and time with the company prior to becoming CEO ("non-CEO tenure") are almost identical to their counterparts for S&P 500 firms during the same time frame.

5.2 Empirical Approach

Equation (25) predicts a positive relation between cash-to-stock and credit spreads in the cross-section even after controlling for leverage ratios and other firm-CEO characteristics. Linearized versions of this equation where CDS rates replace credit spreads are the subject of our empirical investigation.²⁶ More specifically, we examine the prediction that the coefficient on cash-to-stock is positive in the linear relation

$$\ln (CDS)_{it} = \beta_0 + \beta_1 \ln \left(\frac{A}{pS}\right)_{it} + \beta_2 \left(\frac{L}{S+L}\right)_{it} + \beta_3 (\text{controls})_{it} + \epsilon_{it}$$
(27)

where ϵ_{it} is mean zero noise.²⁷

To calculate empirical proxies for our main variable, cash-to-stock, information on each of its three components (A, p, S) is required. We identify S with the current market capitalization of the firm's outstanding common stock. The variable A represents the minimum pay of a manager, and a natural proxy for this component of CEO compensation is salary. When

 $^{^{24}{\}rm The}~{\rm G}$ index is calculated using information on 24 antitakeover provisions. We thank Andrew Metrick for making this data available at the website http://www.som.yale.edu/faculty/am859/.

²⁵We thank John Graham for making this data available at the website http://www.duke.edu/ jgraham. ²⁶Consistent with the fact that a CDS is an insurance against credit risk, it can be shown that the CDS rate is an increasing function of credit spreads in the context of the R model. As a result, CDS rates can also be expressed as $CDS = g\left(\frac{A}{pS_0^R}, \frac{L}{S_0^R+L}, \alpha, \gamma, \tau, r, T\right)$ for a function g whose partial derivatives have the same sign as the function f in equation (25).

²⁷The logarithmic transforms mitigate the effects of right skewness in the CDS rate and compensation data.

CEO compensation does not include options the CEO's proportional holdings of common stock provide an appropriate measure of p. When options are present, they are optimally exercised only when in-the-money. They therefore have no impact on minimum pay A but can have a significant impact on stock ownership. To account for this effect, we make use of effective stock ownership p defined in Section 5.1.

The remaining terms in equation (27) are leverage ratio and controls for the parameters $(\alpha, \gamma, \tau, r, T)$ in the R model. In all regressions we include year fixed effects, so we do not proxy for the risk-free rate r which is constant in the cross-section. Controlling for T is likely not critical since we make use of only five-year CDS rates. The parameter τ in the R model represents the marginal corporate tax rate. To control for cross-sectional variation in this parameter, we utilize the simulated marginal before financing tax rate proxy of Graham (2000).²⁸

In order to proxy for the parameter α in our cross-section of firms it is necessary to account for the presence of idiosyncratic risk. The technical appendix shows that all pricing formulas from the R model are valid in the presence of idiosyncratic risk if the market price of risk α is replaced by the firm specific price of risk α_i . The instantaneous Sharpe ratio of the stock $(\mu_{it}^R - r)/\sigma_{it}^R$, where μ_{it}^R is the instantaneous expected return, identifies α_i . We therefore utilize the Sharpe ratio SR as a control for heterogeneity in the firm-specific price of risk.

We do not utilize a direct proxy for the risk aversion γ of the CEO but argue that omitting risk aversion from regression (27) will bias against finding a positive relationship between cash-to-stock and CDS rates. It is intuitive that for a given compensation contract, more risk averse managers run safer firms and, as a result, spreads are lower. This prediction can be made rigorous in the context of the R model. Managerial risk aversion can also affect compensation terms. In the optimal compensation problem with moral hazard of Garen (1994), more risk averse managers are granted relatively more performance insensitive pay (i.e., higher cash-to-stock). This is because stock pay is more costly to the risk-neutral principal than salary pay when managerial risk aversion is high. Summarizing these two arguments, risk aversion is likely to be positively correlated with cash-to-stock and negatively correlated with CDS rates. Omitting the parameter γ will, therefore, cause the coefficient on cash-to-stock to be biased downward.

5.3 Regression Specification

The parameters A and p in the R model are fixed over the tenure of the manager and, as a result, the empirical predictions from the R model are inherently cross-sectional. To assess equation (27) we therefore interpret our CEO-firm panel dataset as providing a large cross-section. To account for the possibility of correlated errors from our pooled OLS

²⁸Before financing marginal rates are relevant in the context of the R model because interest tax shields are assumed to be deducted from the unlevered LPP.

regressions we cluster at the CEO-firm level.²⁹ In all regressions, the dependent variable is the logarithm of the average of month-end CDS rates for each CEO-firm within each calender year 2001-2006. Our benchmark regression is

$$\ln (CDS)_{i,t} = \beta_0 + \beta_1 \ln (A/(pS))_{i,t} + \beta_2 (L/(S+L))_{i,t} + \beta_3 (SR)_{i,t} + \beta_4 \tau_{i,t} + \beta_5 \ln (Sales)_{i,t-1} + \beta_6 \ln (1 + ROA)_{i,t-1} + \beta_7 \ln (1 + r)_{i,t-1} + \beta_8 \ln (B/M)_{i,t-1} + \beta_9 (Div Dummy)_{i,t-1} + \beta'_{10} (Year Dummy)_{i,t} + \epsilon_{i,t}.$$
(28)

The logarithm of the ratio of CEO salary to effective shareholdings $A/(pS)_{i,t}$, the leverage ratio $L/(S+L)_{i,t}$, the Sharpe ratio $SR_{i,t}$, and the marginal tax rate $\tau_{i,t}$ are from the end of the calendar year preceding the calendar year in which CDS rates are observed. Our timing convention ensures that this information is available to investors when CDS rates are set.

Regression (28) includes controls for factors outside the R model. Sales, ROA, equity return, book-to-market, and an indicator for dividend payment represent potential economic determinants of both compensation terms (e.g., Core et. al. (1999), Fenn and Liang (2001)) and credit spreads (e.g., Campbell and Taksler (2003), Ortiz-Molina (2006)). To isolate the direct impact of cash-to-stock in our regressions, these variables are lagged by one year relative to the date at which CEO salary and stock holdings are measured. Year dummy variables are also included in all regressions to account for, among other effects, timevariation in default risk premia (Berndt et. al. (2005)) and possible trends in CEO pay terms (Gabaix and Landier (2008), and Murphy and Zábojník (2006)).

5.4 Results

Table II presents the results from regression (28) in column (i) and other related specifications in columns (ii)-(vii). The statistically significant positive coefficient on cash-to-stock in all regression specifications provides empirical evidence for the positive association between cash-to-stock and CDS rates formalized in Proposition 3. The economic significance of cash-to-stock is also strong. The logarithms of both variables are used in the Table II regressions, so the regression coefficient on $\ln(A/(pS))$ represents the percentage change in CDS rates that result from a one percent change in cash-to-stock. This coefficient ranges from 0.038 to 0.060, so a change in A/(pS) from the 10th percentile to the 90th percentile would result in a change in CDS rates on the order of 13% to 21%.

The coefficient on leverage ratio is also positive in all regressions, consistent with the prediction of Proposition 3 and with the typical prediction of structural models (e.g., Merton (1974)). The signs of the coefficients on Sales, ROA, stock return, and book-to-market in column (i) are consistent with those reported in studies of corporate bond spread determinants (e.g., Campbell and Taksler (2003), Daniel, Martin, and Naveen (2005), and

²⁹Our standard errors therefore allow for the possibility of correlation among the regression errors for each CEO-firm time-series in our panel. An alternative is to ignore CEO turnover and cluster at the firm level. Both levels of clustering produce similar standard errors.

Ortiz-Molina (2006)). The marginal tax rate is negatively related to CDS rates. This finding is inconsistent with the R model (and the M model) where a higher tax rate τ leads to higher spreads. A possible explanation for the negative sign on the marginal tax rates is that it is positively correlated with profitability and less noisy than ROA. The negative coefficient on the dividend dummy variable is consistent with the interpretation that dividend payers are less financially constrained than dividend non-payers (e.g., Hennessy and Whited (2007)).

Column (ii) reports coefficients from an empirical specification in which the cash-tostock variable is replaced by stock volatility. This regression is motivated by the Merton (1974) model and is similar to specifications considered by Campbell and Taksler (2003). As expected, the coefficient on stock volatility is positive and significant. Column (iii) presents a regression that includes both cash-to-stock and stock volatility. The coefficient on cash-tostock remains strongly significant in this regression, indicating that the R model provides incremental insight relative to the Merton model for pricing default sensitive securities.

5.5 Endogeneity

The coefficient on cash-to-stock in regression (28) will be consistently estimated using pooled OLS only if endogeneity is not an issue, that is only if the error is uncorrelated with the cash-to-stock measure (i.e., $E(\ln(A/(pS))_{it}\epsilon_{it}) = 0$). An important potential source of endogeneity is the omission of a variable that jointly impacts CDS rates and cash-to-stock. Although our controls in regression (28) were chosen to minimize the likelihood of bias, we consider in this subsection endogeneity due to other additional economically motivated variables.

CEO Turnover. CEO turnover may jointly impact CDS rates and cash-to-stock. Clayton, Hartzell, and Rosenberg (2005) document that, due to learning of managerial skill, CEO turnover leads to an increase in a firm's equity volatility. Turnover should therefore reduce bond values, a conjecture that is supported by Adams and Mansi (2008) in a sample of US firms. CEO turnover is also likely to impact our measure of cash-to-stock. Newly hired CEOs have not accumulated stock or option grants and this will mechanically lower their cashto-stock relative to CEO who have been in place for a longer period. Consistent with this intuition, unreported results show a positive association between turnover and cash-to-stock in our sample. These arguments show that omitting turnover from regression (28) can lead to a positive bias in the cash-to-stock coefficient. Column (iv) of Table II shows, however, that this coefficient remains both statistically and economically positive after including turnover (New CEO) as a control.

Corporate Governance. Corporate governance in general, and more specifically antitakover provisions, may be responsible for the joint determination of CEO pay terms and credit spreads. Antitakover provisions may mitigate the disciplining function of the labor market on the risk averse manager. Bertrand and Mullainathan (2003) provide empirical evidence that such managers can choose to lead a "quiet life" by reducing firm risk, and increasing white-collar pay. A reduction in cash flow variability and bankruptcy probability causes credit spreads and CDS rates to be low. In fact, Klock, Mansi and Maxwell (2005) provide cross-sectional evidence that increased antitakeover protection leads to lower credit spreads. CEOs who are sheltered from the market for corporate control may also have the power to influence CEO pay terms. Fahlenbrach (2008) shows that CEOs at firms that are relatively more insulated from the threat of takeover have high levels of pay but low fractional ownership (i.e., they have high cash-to-stock). Ignoring the impact of antitakeover provisions may, thus, lead to a downward bias in the coefficient on cash-to-stock. To address this possibility, we add to regression (28) the Gompers, Ishii, and Metrick (2003) "G-index". Column (iv) of Table II shows that the magnitude of the cash-to-stock coefficient remains statistically and economically significant in the presence of the G-index.³⁰

Exogenous Firm Risk. In a setting where, unlike in the R model, firm risk is exogenous and constant, it is conceivable that the volatility of the firm's underlying assets is related to both cash-to-stock and CDS rates. Garen (1994) argues that in the context of a simple optimal contracting model pay-performance sensitivity should be decreasing in firm risk and salary should be increasing. Aggarwal and Samwick (1999) show empirically that executive pay-performance sensitivity is decreasing in stock volatility. In unreported results, we verify that cash-to-stock is positively correlated with equity volatility in our sample. In the Merton (1974) model credit spreads are an increasing function of underlying asset risk. Campbell and Taksler (2003) provide empirical support for this relationship in a sample of US firms, and the regression reported in Column (ii) of Table II showed this to be the case for CDS rates in our sample. We conclude that omitting proxies for risk can induce an upward bias in the coefficient on cash-to-stock. This is confirmed by regression (iii) in Table II where we include stock volatility as a direct proxy for firm risk. The table shows, however, that the coefficient on cash-to-stock remains positive and significant.

Industry Fixed Effects. In an attempt to control for unobserved omitted variables, we consider augmenting regression (28) with industry fixed effects. Such specifications account for potential endogeneity due to industry-specific, time-invariant components in the errors. As an example, suppose that industries differ with respect to the liquidity of firms' real assets in bankruptcy (e.g., Shleifer and Vishny (1992)). All else equal, firms in high liquidity industries are likely to have lower CDS rates, due to the higher recovery rates for bondholders. Such industries may, for related reasons, also provide CEOs with better outside options. An optimal compensation contract that internalizes the benefit to managers of outside options is likely to include relatively less explicit safe pay and more risky pay, leading to lower cash-to-stock. In this setting, omission of an asset liquidity proxy would lead to an upward bias in the coefficient on cash to stock. We make use of industry fixed effects

³⁰In unreported results, we find that in our sample higher levels of the G-index are associated with lower CDS rates and higher cash-to-stock. This confirms the findings of Klock, Mansi, and Maxwell (2005) and Fahlenbrach (2008). We find that in the subsample of 2305 CEO-firm-years used to undertake the regression reported in Column (vi) of Table II, dropping the G-index produces a lower coefficient on cash-to-stock. This unreported result confirms the economic argument for a downward bias when antitakeover provisions are ignored.

to serve this purpose and at the same time control for other unobserved industry-specific sources of endogeneity.³¹ Regressions (v) and (vi) in Table II re-estimate the coefficients reported in columns (i) and (iv) of Table II with the inclusion of industry fixed effects. The coefficients reported in the first row of the table are generally less positive than their counterparts in columns (i) and (iv) as would be expected if the asset liquidity effect is at play in our sample. These results show that cash-to-stock nevertheless remains as a significant positive determinant of CDS rates.

5.6 Robustness

Table III addresses the robustness of our empirical results.

5.6.1 Alternative specifications

As an alternative to pooled OLS, we apply the Fama-Macbeth technique which identifies the coefficients in regression (28) using the cross-sectional variation in the CEO-firm panel yearby-year. Column (i) of Table III shows the coefficient on cash-to-stock remains statistically significant and is comparable to those reported in Table II. This finding suggests that the coefficient from our pooled OLS regressions are identified by the cross-sectional variation in our sample.

Column (ii) of Table III presents point estimates for the coefficients from regression (28) estimated using a standard regression technique that is robust to outliers.³² The coefficient on cash-to-stock remains positive and of comparable magnitude to the OLS coefficient in Column (i) of Table II.

In column (iii) of Table III we utilize an alternative method to account for managerial option holdings q. In many cases, for example when option holdings are high relative to stock holdings, it can be shown that a version of the R model in which CEOs hold options is isomorphic to a version of the R model with no options but where the proportional stock ownership is increased to $p_s + q$. In addition, the R model must be augmented to increase the bankruptcy threshold V_b by an amount that depends on the ratio $q/(p_s + q)$ and on the (scaled) exercise price of the options K/S. Within this equivalent model, our main prediction that, *ceteris paribus*, an increase in cash-to-stock ratio results in an increase in the CDS rate remains valid. To account for option holdings, we therefore modify our empirical model to include the measures $A/[(p_s + q)S]_{i,t}, q/(p_s + q)_{i,t}, and <math>(K/S)_{it}$. Column (iii) of Table III shows that with this modified measure of cash-to-stock $A/[(p_s + q)S]$, the positive impact on CDS rates is strengthened.

In column (iv) of Table III we proxy for A using the sum of bonus and salary, rather

³¹Each CEO-firm in our sample is assigned into one of twelve Fama and French (1997) industry categories using the four-digit SIC code. We thank Ken French for making the mapping to industry is available on his website http mba.tuck.dartmouth.edupagesfacultyken.frenchdata_library.html.

 $^{^{32}}$ We use SAS proc robustreg to implement the standard M estimation introduced by Huber (1973). This procedure does not allow for clustering of errors, thus the t-statistics from this regression are not comparable to those from our benchmark regression.

than salary on its own. The coefficient on A/(pS) becomes both statistically and economically weaker in this regression. This result shows the importance of excluding performance sensitive pay from the variable A, consistent with the assumptions of the R model.

We have also considered alternative regression specifications with reasonable variation in timing conventions (e.g., by using lagged rather than contemporaneous leverage ratio) and with a variety of subsets of the controls. Such variations have no significant impact on our results.

5.6.2 A broader interpretation of the variable A

Columns (iv)-(vi) present results from regressions that permit an economic interpretation of minimum pay A in the R model that is broader than CEO salary used in regression (28). This interpretation is motivated by economically important mechanisms in the CEO labor markets that suggest a role for CEO age, tenure, and non-CEO tenure to measure A.

First, Bebchuk and Jackson (2005) consider CEO pensions to be like holdings of treasury bonds, emphasizing cases in which CEO pensions are treated as senior claims in bankruptcy or are made inaccessible to the firm's outside security holders using various legal means (see also Bebchuk and Fried (2004) pp. 101-102).³³ The CEOs pension can, therefore, be interpreted as a lower bound on pay. The variable A would then ideally be extracted from information on the value of the pension payments; however, pension disclosure is limited and standard sources (e.g., ExecuComp) do not report this information for our sample. Pension payments are typically calculated using firm-specific formulas based on years of service and cash compensation, so in addition to cash-to-stock we add to regression (28) the CEO's tenure to capture its impact on pension value. Age is also included in this regression since for any given tenure and cash-to-stock the value of the pension is higher when retirement is nearer.

Second, earnings generated outside the firm after date T may be an important component of CEO wealth. Human capital can, therefore, be interpreted as contributing to the variable A. This consideration generates two testable predictions linking two CEO characteristics to CDS rates: (i) All else equal, the variable A is likely to be relatively high for young CEOs, since they have longer labor income duration at date T than old CEOs. (ii) All else equal, managers who invest relatively more in general managerial skill, as opposed to firm-specific skill, will have human capital that is more valuable outside the firm (Murphy and Zábojník (2004)) and, hence, higher A. Following Murphy and Zábojník (2006), we proxy for investment in firm specific skill using the time spent with the firm prior to becoming CEO.

The pension and human capital hypotheses suggest that, in addition to salary, the variables CEO age, tenure, and non-CEO tenure should affect A in the cross-section and, in turn, CDS rates. The pension hypothesis predicts that when added to regression (28), CEO age and tenure should have positive coefficients. The human capital hypothesis predicts that

³³Similarly, Sundaram and Yermack (2007) consider defined benefit pensions to be "inside debt".

CEO age and non-CEO tenure should have negative coefficients. Table III presents results from such regressions in our overall sample (column (v) - all), in a subsample of CEOs whose age is 60 or higher (column (vi) - old), and in a subsample of CEOs whose age is below 60 years (column (vii) - young). Comparing columns (i) of Table II and (v) of Table III shows that including CEO age, tenure, and non-CEO tenure as predictive variables leads to an increase in the magnitude of the cash-to-stock coefficient. Consistent with the pension hypothesis, tenure has a positive and significant coefficient. Age has a significant negative sign, however, which might indicate that for the entire sample human capital considerations are more important to CEOs than pensions. This interpretation is further supported by the significantly negative coefficient on non-CEO tenure.

The old and young CEO subsamples allow us to examine whether the pension hypothesis is more relevant than the human capital hypothesis for CEOs that are near retirement. In the old CEO sample (column (vi)), the coefficient on cash-to-stock increases relative to its estimate in the full sample and the coefficient on non-CEO tenure is less negative. Notably, the coefficient on age changes sign to become weakly positive. Although the coefficient on tenure is only weakly positive, the regression coefficients in column (vi) suggest a link between the value of pensions and CDS rates for firms employing CEOs aged 60 and higher. In the young CEO sample (column (vii)), the coefficient on cash-to-stock is slightly lower than in the full sample, the coefficient on age is more significantly negative, the coefficient on tenure is more positive, and the coefficient on non-CEO tenure is more negative. Overall, these coefficients indicate that human capital may play an important role in determining CDS rates at firms run by younger CEOs.

5.7 Joint determination of leverage and CDS rates.

Section 4 describes the optimal leverage choice in the first stage problem. As Panel B of Figure 6 illustrates, this model predicts that in a cross-section of firms that are optimally levered, there is a positive relationship between cash-to-stock and CDS rates. In this relationship leverage is not held constant across firms, but is also determined by the manager's cash-to-stock. This suggests that cash-to-stock and leverage ratios may be collinear, in which case including both variables in the covariate set will bias the coefficient on cash-to-stock. In column (vii) of Table II we report the coefficients from a model which drops the leverage ratio control from regression (28). This change causes the coefficient on cash-to-stock to increase relative to its value in column (i), suggesting a positive correlation between cash-to-stock and leverage ratios.

In Table IV we present direct evidence on the relationship between cash-to-stock and leverage ratios in our sample. We utilize the same controls as in Graham and Mills (2008), which are similar to those used in Coles, Daniel, and Naveen (2006) and Lewellen (2006). In all regressions, the coefficients on these controls have signs similar to those reported in Coles, Daniel, and Naveen (2006) and Lewellen (2006). Consistent with the first stage model, the regressions in Columns (i)-(iv) of Table IV show that cash-to-stock is a strong positive predictor of leverage ratios. The regressions in columns (v)-(viii) allow leverage ratios to be quadratic in cash-to-stock. The signs on both the cash-to-stock and its square are significantly positive in these regressions, indicating that leverage ratios are convex in cash-to-stock. The downward sloping portion of the quadratic occurs only at very low, and empirically rare, cash-to-stock levels. Thus, there is not strong evidence in our data for the U-shaped relationship between cash-to-stock and leverage ratios depicted in Panel A of Figure 6. This raises the possibility that forces outside the model are at play. One possibility is that the assumption that managers can perfectly control risk is not valid for firms run by managers having low cash-to-stock ratios. Another possibility that in selecting optimal compensation terms, only high level of cash-to-stock are optimal.

6 Conclusion

In this paper, we demonstrate the relevance of the agency costs of Jensen and Meckling (1976) for structural models of leverage choice and credit spreads. Assuming a realistic compensation structure for risk-averse managers, consisting of cash and stock, we show that managers will optimally choose to lever the firm and that their resulting pay will be convex in the firm's terminal liquidating pre-tax payout. This convexity induces asset substitution, leading to riskier payouts and higher credit spreads than predicted by the prior literature. We also demonstrate that optimal leverage choice is the result of a balance between tax benefits and the utility cost of ex-post risk shifting. Our work thus highlights that operating behavior induced by compensation terms can be of first-order importance for understanding debt levels and credit spreads.

To empirically evaluate our model, we use a large cross-section of 608 US based corporate credit default swaps covering 2001-2006. We confirm the model's prediction of a significantly positive relationship between cash-to-stock ratios and CDS rates when CEO salary is used as a proxy for cash pay and CEO stock holdings are used as proxies for stock pay. This result is robust to including option-based pay and to alternative interpretations of cash as pension or implicit compensation. Reasonable variation in proxies for cash-to-stock result in variation in CDS rates of up to 21% even after controlling for the traditional structural determinants of spreads such as leverage and stock volatility. We further find that CEO cash-to-stock is informative for leverage choice. We interpret our findings as supportive of prediction that cross-sectional differences in CEO compensation terms provide economically important information for pricing default-sensitive securities and predicting leverage ratios. Because cash-to-stock and capital structure can be jointly determined as an outcome of an optimization process of the firm value and its allocation among various claimants, our empirical results are not necessarily causal. The exploratory empirical associations that we document in this paper should, however, help guide further empirical research that more directly addresses potential endogeniety in the data.

Our model can be extended in many interesting directions. One possibility is to consider

the optimal contract from the shareholders' perspective. This would require that we consider one more prior optimization problem in which the amount of cash compensation and the number of unlevered shares are determined, subject to a participation constraint. Addressing this problem may shed light on our empirical finding that cash-to-stock in the data is set to levels where leverage is an increasing, rather than U-shaped function.

A more ambitious extension would place our model in a dynamic context. The model in this paper could be considered one stage of a multi-period problem in which periodic capital structure and default decisions could be made. Dynamic contracting as in DeMarzo and Sannikov (2006) could also be incorporated. These changes, if analytically tractable, would add significant realism and allow for empirical tests of a dynamic capital structure model in which risk shifting plays an important role.

Appendix

Proofs of Propositions 1, 2 and 3

Before proceeding with the proofs, we give a preliminary lemma.

Lemma 5 Fix the parameters (A, p, γ, τ) , and define the tangency point $\overline{V} \in (V_b, \infty)$ for any $V_b > 0$ as the unique solution to the nonlinear equation (15) where V_b^R is replaced by V_b . The variable \overline{V} may be expressed as a function of two variables $\overline{V} = \overline{V}\left(\frac{A}{p(1-\tau)}, V_b\right)$ where

$$\frac{\partial \overline{V}}{\partial V_b} > 1, \quad \frac{\partial \overline{V}}{\partial A} > 0, \quad \frac{\partial \overline{V}}{\partial p} < 0.$$

Proof. Dividing equation (15) by $(p(1-\tau))^{1-\gamma}$ shows that only the variables $A/p(1-\tau)$ and V_b are relevant for defining the tangency point \overline{V} . Multiplying equation (15) by \overline{V} and differentiating it with respect to V_b gives

$$A + p(1-\tau)(\overline{V} - V_b) = \gamma p(1-\tau)\overline{V}(\frac{\partial \overline{V}}{\partial V_b} - 1).$$

Recalling that $\overline{V} > V_b$, we therefore have $\frac{\partial \overline{V}}{\partial V_b} > 1$.

Given that \overline{V} depends on the ratio $A/(p(1-\tau))$ we see that $\frac{\partial \overline{V}}{\partial A} > 0$ is equivalent to $\frac{\partial \overline{V}}{\partial p} < 0$. We shall prove that $\frac{\partial \overline{V}}{\partial p} < 0$. Without loss of generality, we assume that A = 1 and $\tau = 0$ and rewrite equation (15) as

$$\left[1+p(\overline{V}-V_b)\right]^{\gamma}-\left[1+\gamma p(\overline{V}-V_b)\right]=p(\gamma-1)\overline{V}.$$
(29)

Notice that when $\gamma = 2$

$$\overline{V} = V_b + \sqrt{\frac{V_b}{p}}$$

which is decreasing in p. To proceed for an arbitrary $\gamma > 0$, differentiate equation (29) with respect to p, holding V_b constant, and substitute again from equation (29) to arrive at

$$C + \frac{\partial \overline{V}}{\partial p}B = 0 \tag{30}$$

where

$$B = \gamma \left([1 + p(\overline{V} - V_b)]^{\gamma - 1} - 1 \right),$$

and

$$C = \frac{1}{p^2} \left(1 - [1 + p(\overline{V} - V_b)]^{\gamma} + \gamma p(\overline{V} - V_b) [1 + p(\overline{V} - V_b)]^{\gamma - 1} \right)$$

It can be checked that B is non-negative when $\gamma \geq 1$ and non-positive when $\gamma \leq 1$. To determine the sign of C, we may rewrite $C = F(p(\overline{V} - V_b))/p^2$, where the function F is defined by

$$F(x) = \gamma x (1+x)^{\gamma-1} + 1 - (1+x)^{\gamma}.$$

Since F(0) = 0 and $F'(x) = \gamma(\gamma - 1)x(1 + x)^{\gamma - 2}$, we conclude that $C \ge 0$ if $\gamma \ge 1$ and $C \le 0$ if $\gamma \le 1$. Referring to equation (30) we conclude that $\frac{\partial \overline{V}}{\partial p} < 0$ for all values of γ .

Proof of Proposition 1: The proof proceeds in three steps. In the first, we define the Legendre-Fenchel (L-F) transform, a useful tool from convex analysis, and provide some

simple geometric intuition that helps to understand the basic principles underlying this operator. Readers familiar with convex analysis can skip this step. Second, using the L-F transform, we characterize the manager's choice of LPP, V_T , when he takes any bankruptcy threshold V_b as given. In the third step, we prove the existence of a unique V_b^R which solves the bankruptcy threshold condition (6) and is simultaneously consistent with the optimal LPP choice, V_T^R .

Step 1: The function $h^* : \mathbb{R} \longrightarrow \mathbb{R} \cup \{\infty\}$ denotes the L-F transform of h defined by

$$h^*(x^*) = \sup_{x \in \mathbb{R}} \left(xx^* - h(x) \right).$$
(31)

For additional mathematical detail see Chapter 3 of Ekeland and Turnbull (1983) or, for a more abstract treatment, Rockafellar (1970). Similarly, the function $h^{**} : \mathbb{R} \longrightarrow \mathbb{R} \cup \{\infty\}$ denotes the double Legendre-Fenchel transform of h defined as

$$h^{**}(x) = \sup_{x^* \in \mathbb{R}} \left(x^* x - h^*(x^*) \right).$$
(32)

We will use two basic results from convex analysis.

- R1 The function h^{**} is the convex envelope of h; that is, the function h^{**} is the largest convex function dominated by h.
- R2 If h^* admits a supporting line³⁴ at $x^* \in \mathbb{R}$ with slope k, then h^{**} admits a supporting line at k with a slope x^* .

For a geometric intuition supporting these results, see the technical appendix that accompanies this paper.

Step 2: In this step, we reformulate Theorem 1 of Carpenter (2000) when V_b is fixed. Here, we provide a new proof that applies the L-F transform techniques and generalizes Carpenter's Theorem to a larger class of compensation schedules. This may be useful, for example, if the compensation function depends on more than one variable.

Let us first, define the function $\varphi : \mathbb{R} \longrightarrow \mathbb{R} \cup \{\infty\}$ by

$$\varphi(x) = \begin{cases} -\frac{\left(A + p(1-\tau)(x-V_b)^+\right)^{1-\gamma}}{1-\gamma} & \text{if } x \ge 0, \\ +\infty & \text{otherwise.} \end{cases}$$

Define also the tangency point $\overline{V} \in (V_b, \infty)$ as the unique solution to the nonlinear equation

$$\left[\frac{[A+p(1-\tau)(\overline{V}-V_b)]^{1-\gamma}}{1-\gamma} - \frac{A^{1-\gamma}}{1-\gamma}\right]\overline{V}^{-1}$$
$$= p(1-\tau)\left[A+p(1-\tau)(\overline{V}-V_b)\right]^{-\gamma} \equiv -\varphi'(\overline{V}).$$
(33)

³⁴The function h^* admits a supporting line at x^* if there exists $k \in \mathbb{R}$ such that

$$h^*(z) \ge h^*(x^*) + k(z - x^*)$$

for all $z \in \mathbb{R}$. We will say then that h^* admits a supporting line at x^* with slope k. The geometric meaning of a supporting line is intuitive: the supporting line must be uniformly below the graph of the function h^* . For instance, a convex and differentiable function admits a supporting line at any point and the supporting line, in this case, is just the familiar tangent line. Supporting lines are also called subdifferentials.

Direct computations give the analytical formula for the L-F transform of φ

$$\varphi^*(x^*) = \begin{cases} +\infty & \text{if } x^* > 0, \\ x^* \left(V_b - \frac{A}{p(1-\tau)} \right) + \frac{\gamma}{1-\gamma} \left(\frac{-x^*}{p(1-\tau)} \right)^{-\frac{1-\gamma}{\gamma}} & \text{if } x^* \in [\varphi'(\overline{V}), 0], \\ -\varphi(0) = \frac{A^{1-\gamma}}{1-\gamma} & \text{if } x^* \le \varphi'(\overline{V}). \end{cases}$$

The function φ^* is differentiable on $(-\infty, \varphi'(\overline{V})) \cup (\varphi'(\overline{V}), 0)$ with a derivative

$$\nabla \varphi^*(x^*) = \begin{cases} \left(V_b - \frac{A}{p(1-\tau)} \right) + \frac{1}{p(1-\tau)} \left(\frac{-x^*}{p(1-\tau)} \right)^{-\frac{1}{\gamma}} & \text{if } x^* \in (\varphi'(\overline{V}), 0), \\ 0 & \text{if } x^* < \varphi'(\overline{V}). \end{cases}$$

Furthermore, at the point $x^* = \varphi'(\overline{V})$, the function φ^* has many supporting lines with slopes in the interval $[0, \overline{V}]$ since \overline{V} is the right derivative of φ^* at $\varphi'(\overline{V})$.

Finally, the double L-F transform φ^{**} can also be derived and is given by

$$\varphi^{**}(x) = \begin{cases} +\infty & \text{if } x < 0, \\ -\frac{A^{1-\gamma}}{1-\gamma} - p(1-\tau) \left(A + p(1-\tau)(\overline{V} - V_b)\right)^{-\gamma} x & \text{if } x \in [0, \overline{V}] \\ \varphi(x) & \text{otherwise.} \end{cases}$$

Now, the second stage optimization problem (13) may be equivalently formulated as³⁵

$$\mathcal{P}: \quad \inf_{V_T} E\left[\varphi(V_T)\right],\\ \text{subject to } E(\xi_T V_T) \le V_0.$$

The problem \mathcal{P} is not standard because we minimize a non-convex function. The strategy of the proof is as follows: We first solve a "convexified" version of problem \mathcal{P} defined as

$$\tilde{\mathcal{P}}: \quad \inf_{V_T} E\left[\varphi^{**}(V_T)\right],\\ \text{subject to } E(\xi_T V_T) \le V_0, \end{cases}$$

and then show that the optimum for $\tilde{\mathcal{P}}$ is actually an optimum for \mathcal{P} .

The problem $\tilde{\mathcal{P}}$ is convex since the function φ^{**} is convex (this is an implication of the result R1). So, by standard results from optimization (Luenberger (1969)) the first order conditions

$$\nabla \varphi^{**}(V_T) = -\lambda \xi_T, \quad \lambda > 0 \text{ is such that } E(\xi_T V_T) = V_0,$$

are both necessary and sufficient. We see then that the first order conditions for problem P stipulate that φ^{**} admits a supporting line at V_T with slope $-\lambda\xi_T$ (almost surely). Given the supporting line duality result R2, we see that the first order conditions for problem \tilde{P} may be restated by saying that φ^* admits a supporting line at $-\lambda\xi_T$ with slope V_T (almost surely). Given the closed form expression of φ^* we see that the first order conditions for \tilde{P} can be expressed as³⁶

$$V_T = \begin{cases} \left(V_b - \frac{A}{p(1-\tau)} \right) + \frac{1}{p(1-\tau)} \left(\frac{\lambda \xi_T}{p(1-\tau)} \right)^{-\frac{1}{\gamma}} & \text{if } -\lambda \xi_T > \varphi'(\overline{V}), \\ 0 & \text{if } -\lambda \xi_T \le \varphi'(\overline{V}), \end{cases}$$

³⁵Note that the positivity constraint is now incorporated in the definition of φ and that the maximization has been replaced with a minimization because φ reverses the sign of the utility function.

³⁶When $-\lambda\xi_T = \varphi'(\overline{V})$ the first order conditions mandate that V_T could take any value in the interval $[0, \overline{V}]$ because φ^* has multiple supporting lines at $\varphi'(\overline{V})$. We selected the value 0 for V_T in the state $-\lambda\xi_T = \varphi'(\overline{V})$. This choice will have no impact on the final utility since the probability of the event $\{-\lambda\xi_T = \varphi'(\overline{V})\}$ is just 0 given the lognormal distribution of ξ_T .

with the budget restriction $E(\xi_T V_T) = V_0$. Substituting $\overline{\xi} = -\frac{\varphi'(\overline{V})}{\lambda}$ in the above expression gives the optimal LPP for problem $\tilde{\mathcal{P}}$

$$V_T = \left[\overline{V} + \left(\overline{V} - V_b + \frac{A}{p(1-\tau)}\right) \left(\left(\frac{\xi_T}{\overline{\xi}}\right)^{-\frac{1}{\gamma}} - 1\right)\right] \mathbf{1}_{\{\xi_T \le \overline{\xi}\}}.$$
 (34)

The last step is to establish that the optimal V_T for $\tilde{\mathcal{P}}$ is also optimal for \mathcal{P} . To see this, observe first that φ and φ^{**} take on the same value on the set $\{0\} \cup [\overline{V}, +\infty]$. Therefore, since V_T takes values only on the set $\{0\} \cup [\overline{V}, +\infty]$, we have

$$E\left[\varphi(X) - \varphi(V_T)\right] = E\left[\varphi(X) - \varphi^{**}(V_T)\right] \ge E\left[\varphi^{**}(X) - \varphi^{**}(V_T)\right]$$

for any feasible X satisfying $E(\xi_T X) = V_0$. Now, from the first order conditions of Problem $\tilde{\mathcal{P}}$, we know that φ^{**} admits a supporting line at V_T with slope $-\lambda\xi_T$. Consequently, from the definition of supporting lines,

$$E\left[\varphi^{**}(X) - \varphi^{**}(V_T)\right] \ge -E\left[\lambda\xi_T\left(X - V_T\right)\right] = 0$$

thereby proving the optimality of V_T , defined by equation (34), for Problem \mathcal{P} .

Step 3: We now show that there exists a unique $V_b^R \in (L,\infty)$ such that equation (6) is satisfied with V_T given by (34). For any $V_b \in [L, \infty)$ define the continuous function

$$\zeta: V_b \to \zeta(V_b) = L + \frac{\tau}{1 - \tau} B_0$$

where B_0 is defined by (5) and where V_T in this formula is given by (34)-(33). Observe that, by definition, $\zeta(L) > L$. We now show that the function ζ is non increasing with respect to V_b and therefore, there exists a unique $V_b^R \in (L, \infty)$ such that $\zeta(V_b^R) = V_b^R$. It can be shown (see the discussion in Section 3.2) that the special form of V_T given in

(34) implies that bond values are given by

$$B_0 = Le^{-rT} \mathcal{N}(d(0, 1, 1/\overline{\xi}))$$

where the function d is defined by

$$d(t, x, m) = \left(-\ln m + r(T - t) - \frac{\alpha^2}{2}(1 - 2(1 - x))(T - t)\right) / (\alpha \sqrt{T - t}).$$

The function d is non increasing with respect to m and, therefore, B_0 is non decreasing with respect to $\overline{\xi}$. As a result, to establish that ζ is decreasing in V_b we need to prove that $\overline{\xi}$ is non increasing with respect to V_b . To this end, one can explicitly compute the expectation in the constraint $V_0 = E(\xi_T V_T)$ which defines $\overline{\xi}$ to obtain

$$V_{0} = \left(\overline{V} - V_{b} + \frac{A}{p(1-\tau)}\right) e^{-(r+\alpha^{2}/(2\gamma))\gamma^{*}T} \overline{\xi}^{1/\gamma} \mathcal{N}(d(0,\gamma^{*},1/\overline{\xi})) \\ + \left(V_{b} - \frac{A}{p(1-\tau)}\right) e^{-rT} \mathcal{N}(d(0,1,1/\overline{\xi})).$$
(35)

The proof of Proposition 2 gives more detail on this computation. Differentiate the above equation with respect to V_b to get the following expression (after some long but straightforward calculations)

$$0 = \left(\frac{\partial \overline{V}}{\partial V_b} - 1\right) e^{-\left(r + \frac{\alpha^2}{2\gamma}\right)\gamma^* T} \overline{\xi}^{\frac{1}{\gamma}} \mathcal{N}(d(0, \gamma^*, 1/\overline{\xi})) + e^{rT} \mathcal{N}(d(0, 1, 1/\overline{\xi})) + \left(\overline{V} - V_b + \frac{A}{p(1 - \tau)}\right) e^{-\left(r + \frac{\alpha^2}{2\gamma}\right)\gamma^* T} \frac{\partial \overline{\xi}^{\frac{1}{\gamma}}}{\partial V_b} \mathcal{N}(d(0, \gamma^*, 1/\overline{\xi})) + \overline{V} \frac{\partial \overline{\xi}^{\frac{1}{\gamma}}}{\partial V_b} e^{-rT} \frac{\gamma}{\alpha\sqrt{T}} \frac{1}{\overline{\xi}^{\frac{1}{\gamma}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{d^2(0, 1, 1/\overline{\xi})}{2}}.$$

This, in turn, allows us to sign the term

$$\frac{\partial \overline{\xi}^{\frac{1}{\gamma}}}{\partial V_b} \left[\left(\overline{V} - V_b + \frac{A}{p(1-\tau)} \right) e^{-(r + \frac{\alpha^2}{2\gamma})\gamma^* T} \mathcal{N}(d(0,\gamma^*, 1/\overline{\xi})) + \overline{V} e^{-rT} \frac{\gamma}{\alpha\sqrt{T}} \frac{1}{\overline{\xi}^{\frac{1}{\gamma}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{d^2(0, 1, 1/\overline{\xi})}{2}} \right] \\ = -\left(\frac{\partial \overline{V}}{\partial V_b} - 1 \right) e^{-(r + \frac{\alpha^2}{2\gamma})\gamma^* T} \overline{\xi}^{\frac{1}{\gamma}} \mathcal{N}(d(0, \gamma^*, 1/\overline{\xi})) - e^{rT} \mathcal{N}(d(0, 1, 1/\overline{\xi})) < 0,$$

and recalling that $\frac{\partial \overline{V}}{\partial V_b} > 1 >$ (see Lemma 5), we deduce that $\frac{\partial \overline{\xi}^{\frac{1}{\gamma}}}{\partial V_b} < 0$. Since $\gamma > 0$ we conclude that

$$\frac{\partial \xi}{\partial V_b} < 0$$

Proof of Proposition 2: The valuation formula

$$V_t = E_t \left[\frac{\xi_T}{\xi_t} V_T \right]$$

together with the dynamic equation for the state price density and the optimal firm value given in (14) give the expression for time t firm value, V_t , in Proposition 2. To see this, observe that conditional on \mathcal{F}_t , $\ln(\xi_T)$ is normally distributed with mean $\ln(\xi_t) - (r + \alpha^2/2)(T - t)$ and variance $\alpha^2(T-t)$. Substituting the expression for V_T in (14), and computing the expectation in the relevant regions yields the expression for V_t as a function of ξ_t . Application of Itô's Lemma to V_t results in equation (17) for time-t volatility.

Proof of Proposition 3:

We fix the R model's parameters $(\alpha, \gamma, \tau, r, t, T)$ and shorten notation by ignoring the dependency upon these variables. We begin this proof with two preliminary lemmas.

Lemma 6 In the context of the R model, credit spreads and the equity value can be represented as

$$\rho_0^R = \rho_0^R \left(A, p, L, V_0 \right), \qquad S_0^R = S_0^R \left(A, p, L, V_0 \right)$$
(36)

with

$$\frac{\partial \rho_0^R}{\partial A} \geq 0, \quad \frac{\partial \rho_0^R}{\partial V_0} \leq 0, \quad \frac{\partial S_0^R}{\partial A} \geq 0, \quad \frac{\partial S_0^R}{\partial V_0} \geq 0.$$

Furthermore, for any x > 0, the R model exhibits the homogeneity property

$$\rho_0^R\left(\frac{A}{px}, 1, \frac{L}{x}, \frac{V_0}{x}\right) = \rho_0^R\left(A, p, L, V_0\right), \qquad S_0^R\left(\frac{A}{px}, 1, \frac{L}{x}, \frac{V_0}{x}\right) = \frac{S_0^R\left(A, p, L, V_0\right)}{x}.$$
 (37)

Proof of Lemma 6: Equation (21), (23) and (19) show that the endogenous variables ρ_0^R and S_0^R depend on the variables $(\overline{\xi}, V_b^R, \overline{V}, V_0)$. The variables $(\overline{\xi}, V_b^R, \overline{V})$ are jointly determined as the solution of a system of three non-linear equations consisting of

$$0 = E\left(\xi_T\left[\overline{V} + \left(\overline{V} - V_b^R + \frac{A}{p(1-\tau)}\right)\left(\left(\frac{\xi_T}{\overline{\xi}}\right)^{-\frac{1}{\gamma}} - 1\right)\right]\mathbf{1}_{\{\xi_T \le \overline{\xi}\}}\right) - V_0, \tag{38}$$

$$0 = L + \frac{\tau}{1 - \tau} E\left[\xi_T L \mathbf{1}_{\xi_T \le \overline{\xi}}\right] - V_b^R,\tag{39}$$

$$\left[\frac{\left[\frac{A}{p(1-\tau)} + (\overline{V} - V_b^R)\right]^{1-\gamma}}{1-\gamma} - \frac{\left(\frac{A}{p(1-\tau)}\right)^{1-\gamma}}{1-\gamma}\right]\overline{V}^{-1} = \left[\frac{A}{p(1-\tau)} + (\overline{V} - V_b^R)\right]^{-\gamma}.$$
 (40)

The equity price and credit spreads can be expressed as in equation (36) since the input variables of the system (38), (39) and (40) are $(A, p.L, V_0)$.

In order to analyze the impact of increasing A on credit spreads and equity price, holding (p, L, V_0) constant, observe that if $(A, \overline{\xi})$ are known, equation (39) can be used to determine the variable V_b^R and the variable \overline{V} can be calculated from equation (40). Thus, equation (38) may be written as $F(\overline{\xi}, A) = 0$ for a given function F. Using the implicit function theorem shows that there exists a differentiable function ρ such that $\overline{\xi} = \rho(A)$ with $\varrho'(A) = -\frac{\partial F/\partial A}{\partial F/\partial \overline{\xi}}$. If we increase A and keep $\overline{\xi}$ constant in equation (38), V_b^R is unchanged and, from Lemma 5, \overline{V} will increase. Therefore, $\partial F/\partial A > 0$. If we increase $\overline{\xi}$ and keep A constant, equation (39) shows that V_b^R increases. Recalling that $\partial \overline{V}/\partial V_b > 1$ (e.g., Lemma 5), both \overline{V} and $\overline{V} - V_b^R$ increase as a result of the increase in V_b^R . Using this result, an inspection of equation (38) shows that $\partial F/\partial \overline{\xi} > 0$. The function ρ is thus decreasing and when A is increased holding constant (p, L, V_0) , the variable $\overline{\xi}$ is smaller. Credit spreads are decreasing with $\overline{\xi}$ (e.g., equation (21)) and as a result, $\partial \rho_0^R / \partial A \ge 0$. Notice also that when the variable A is increased holding constant (p, L, V_0) , the variable $\overline{\xi}$ is smaller, and as result, bond price B_0^R and the threshold V_b^R is decreased (see equation (39)). Inspecting equation (23) at time t = 0, shows that the equity price increases as a result of an increase in A because bond prices and the threshold V_b^R are decreased.

In order to analyze the impact of increasing V_0 on the spreads, holding (A, p, L) constant, observe that equation (38) may be written as $G(\overline{\xi}, V_0) = 0$ for a given function G. Using the implicit function theorem shows that there exists a differentiable function ς such that $\overline{\xi} = \varsigma(V_0)$ with $\varsigma'(V_0) = -\frac{\partial G/\partial V_0}{\partial G/\partial \overline{\xi}}$. We have $\partial G/\partial V_0 = -1$ and, using the same argument as for F we can establish that $\partial G/\partial \overline{\xi} > 0$. We conclude that the function ς is increasing and, because spreads are decreasing in the variable $\overline{\xi}$, they are lower when V_0 is elevated holding constant the variables (A, p, L) which results in $\partial \rho_0^R/\partial V_0 \leq 0$. To asses the impact of an increase in V_0 on equity price, we take the expectation of equation (8) to get the valuation formula

$$S_0^R = E\left(\xi_T\left[\overline{V} - V_b^R + \left(\overline{V} - V_b^R + \frac{A}{p(1-\tau)}\right)\left(\left(\frac{\xi_T}{\overline{\xi}}\right)^{-\frac{1}{\gamma}} - 1\right)\right]\mathbf{1}_{\{\xi_T \le \overline{\xi}\}}\right).$$
(41)

When the LPP V_0 is elevated holding (A, p, L) constant, the variable $\overline{\xi}$ increases, the threshold V_b^R also increases (see equation (39)) and because $\partial \overline{V}/\partial V_b > 1$ (e.g, Lemma 5), $(\overline{V} - V_b^R)$ increases as a result of the increase in V_b^R . Inspecting equation (41) shows then that $\partial S_0^R/\partial V_0 \geq 0$.

We turn to the proof of the homogeneity property (37). Inspecting the system (38), (39) and (40) shows that the variables (A, p) are only relevant through the ratio $\frac{A}{p}$ (recall that

the variable $(1 - \tau)$ is fixed). It can also be checked that the solution of the system (38), (39) and (40) where the input parameters $(A, p.L, V_0)$ are replaced by $\left(\frac{A}{px}, 1, \frac{L}{x}, \frac{V_0}{x}\right)$ is given by $\left(\overline{\xi}, \frac{V_b^R}{x}, \frac{\overline{V}}{x}\right)$. Substituting these values in the pricing equations (21) and (23) gives the homogeneity properties (37).

Lemma 7 Credit spreads from the R model can be expressed as

$$\rho_0^R = m(A, p, L, S_0^R) \tag{42}$$

where m is a function that is increasing in its first argument, A, and its third argument L.

Proof of Lemma 7: Spreads are given by equation (21), and the inspection of this equation shows that in order to establish Lemma 7, we need to establish that $\overline{\xi}$ is decreasing in A and in L. We make the dependence of the credit spread upon the variable S_0^R explicit by observing the variables $(V_b^R, \overline{V}, \overline{\xi})$ can be determined as the solution of the alternative system of equations (41), (39) and (40). The representation (42) can be justified by observing that the input variables for this system are (A, p, L, S_0^R) (instead of (A, p, L, V_0)).

Duplicating the proof of Lemma 6, it can be shown that spreads increase when A is increased with the variables (p, L, S_0^R) held constant.

In order to analyze the impact of increasing A on credit spreads, holding (p, L, S_0^R) constant, observe that if $(A, \overline{\xi})$ are known, equation (39) can be used to determine the variable V_b^R and the variable \overline{V} can be calculated from equation (40). Thus, equation (41) may be written as $F(\overline{\xi}, A) = 0$ for a given function F. Using the implicit function theorem shows that there exists a differentiable function ρ such that $\overline{\xi} = \rho(A)$ with $\rho'(A) = -\frac{\partial F/\partial A}{\partial F/\partial \overline{\xi}}$. If we increase A and keep $\overline{\xi}$ constant in equation (41), V_b^R is unchanged and, from Lemma 5, \overline{V} will increase. Therefore, $\partial F/\partial A > 0$. If we increase $\overline{\xi}$ and keep A constant, equation (39) shows that V_b^R increases. Recalling that $\partial \overline{V}/\partial V_b > 1$ (e.g, Lemma 5), both \overline{V} and $\overline{V} - V_b^R$ increase as a result of the increase in V_b^R . Using this result, an inspection of equation (41) shows that $\partial F/\partial \overline{\xi} > 0$. We conclude that the function ρ is decreasing and, as a result spreads are larger when A is larger, holding constant the variable (p, L, S_0^R) .

In order to analyze the impact of increasing L on the spreads, holding (A, p, S_0^R) constant, observe that equation (41) may be written as $H(\bar{\xi}, L) = 0$ for a given function H. Using the implicit function theorem shows that there exists a differentiable function v such that $\bar{\xi} = v(L)$ with $v'(L) = -\frac{\partial H/\partial L}{\partial H/\partial \bar{\xi}}$. If we increase L and keep $\bar{\xi}$ constant in equation (39), the variable V_b^R is elevated. Lemma 5 shows that \bar{V} will increase by more than V_b^R and as a result, $\partial H/\partial L > 0$. If we increase $\bar{\xi}$ and keep L constant, equation (39) shows that V_b^R increases. Lemma 5 shows then that $\bar{V} - V_b^R$ increases. Using this result, an inspection of equation (41) shows that $\partial H/\partial \bar{\xi} > 0$. We conclude that the function v is decreasing and, as a result spreads are larger when L is larger, holding constant the variables (A, p, S_0^R) .

Turning now to the proof of Proposition 3, we use the homogeneity property (37) in Lemma 6 with $x = S_0^R$ and the result in Lemma 7 to get

$$\rho_t^R = m(A, p, L, S_0^R) = m\left(\frac{A}{pS_0^R}, 1, \frac{L}{S_0^R}, 1, \right) = f\left(\frac{A}{pS_0^R}, \frac{L}{S_0^R + L}\right).$$

We conclude the proof by observing that the function f is increasing in its first (resp. second) argument, $\frac{A}{pS_0^R}$ (resp. $\frac{L}{S_0^R+L}$), because Lemma 7 shows that the function m is increasing in A (resp. in L).

Proof of Proposition 4: Given any $p \in [p_0, 1]$, we know from Proposition 1 that there exist a unique bankruptcy threshold V_b^R (call it $V_b^R(p)$) associated with the solution V_T^R of the R model (13). Equation (11) may be written as

$$p = \psi(p)$$

where the function ψ is defined by

$$\psi(p) = p_0 \left(1 + \frac{1}{\left(\frac{C_0^R}{B_0^R}\right)_p - 1} \right)$$

and where $\left(\frac{C_0^R}{B_0^R}\right)_p$ is the ratio of bond price to the firm cash flow when the manager's compensation includes a proportion p of the firm's equity. Note that by combining these equations, the term $\left(\frac{C_0^R}{B_0^R}\right)_p$ may be written

$$\begin{split} \left(\frac{C_0^R}{B_0^R}\right)_p &= (1-\tau)\frac{V_0}{B_0^R} + \tau (L-B_0^R)\frac{1}{L} \\ &= \tau \frac{V_0}{V_b^R(p)-L} + \tau - (1-\tau)(V_b^R(p)-L)\frac{1}{L}. \end{split}$$

To establish the monotonicity of the function ψ , we must establish the monotonicity of the function $p \to V_b^R(p)$. To this end, we shall analyze how the graph of the function ζ defined in the proof of Proposition 1 changes when p is changed. We will establish in a Lemma at the end of this proof that when p increases, the graph of the function ζ is shifted up and therefore, $V_b^R(p)$ increases (since it is the unique fixed point of ζ). Hence, the function ψ is non-decreasing with respect to p.

To conclude, let us notice that the function ζ is continuous in p since it involves only smooth transformations. Therefore its fixed point $(V_b^R(p))$ is also continuous in p, which in turn implies that the function ψ is continuous in p. Given that the function ψ is non decreasing in p with

$$\psi(p_0) > p_0$$

and

$$\psi(1) = p_0 \left(1 + \frac{1}{\left(\frac{C_0^R}{B_0^R}\right)_{p=1} - 1} \right) < 1$$
(43)

we see that there must exist a unique $p \in (p_0, 1)$ such that

$$\psi(p) = p$$

Lemma 8 The graph of the function ζ shifts up when the equity share parameter p increases.

Proof of Lemma 8: Recall that the function ζ can be expressed as

$$\zeta(V_b) = L + \frac{\tau}{1-\tau} L e^{-rT} \mathcal{N}(d(0, 1, 1/\overline{\xi})).$$

Notice that equation (35) which defines $\overline{\xi}$ can alternatively be written as

$$V_0 = \left(\overline{V} - V_b + \frac{A}{p(1-\tau)}\right) D + \overline{V}e^{-rT}\mathcal{N}(d(0,1,1/\overline{\xi}))$$
(44)

where

$$D = E\left[\xi_T\left(\left(\frac{\xi_T}{\overline{M}}\right)^{-1/\gamma} - 1\right)\mathbf{1}_{\xi_T \leq \overline{\xi}}\right]$$

= $e^{-(r+\alpha^2/(2\gamma))\gamma^*T} \overline{\xi}^{1/\gamma} \mathcal{N}(d(0,\gamma^*,1/\overline{\xi})) - e^{-rT}\mathcal{N}(d(0,1,1/\overline{\xi}))$
> 0.

We begin by differentiating (44) with respect to p while holding V_b constant. After some long but straightforward computation, one can show that

$$\frac{\partial \overline{\xi}}{\partial p} \tilde{D} = \frac{A}{p^2 (1-\tau)} D - \frac{\partial \overline{V}}{\partial p} \left[D + e^{-rT} \mathcal{N}(d(0,1,1/\overline{\xi})) \right]$$

where

$$\begin{split} \tilde{D} &= \left[\frac{1}{\gamma} \overline{\xi}^{\frac{1}{\gamma}-1} e^{-(r+\alpha^2/(2\gamma))\gamma^* T} \left(\overline{V} - V_b + \frac{A}{p(1-\tau)}\right) \mathcal{N}(d(0,\gamma^*,1/\overline{\xi})) \\ &+ \overline{V} e^{-rT} \frac{1}{\overline{\xi}} \frac{1}{\alpha\sqrt{T}} n(d(0,1,1/\overline{\xi})) \right] > 0. \end{split}$$

Lemma 5 establishes that $\frac{\partial \overline{V}}{\partial p} < 0$ and therefore we can conclude that $\frac{\partial \overline{\xi}}{\partial p} > 0$.

References

- Acharya, Viral V. and Timothy C. Johnson. (2006). Insider Trading in Credit Derivatives. Journal of Financial Economics, forthcoming.
- Adams, John, C., and Sattar A. Mansi. (2008). CEO Turnover and bondholder wealth. *Journal of Banking and Finance*, Forthcoming .
- Aggarwal, Rajesh K., and Andrew A., Samwick. (1999). The Other Side of the Trade-Off: The Impact of Risk on Executive Compensation. *The Journal of Political Economy* 107: 65-105.
- Altman, Edward I. (1968). Financial ratios, discriminant analysis, and the prediction of corporate bankruptcy. *Journal of Finance* 23, 589-609.
- Anderson, Ronald W., Sattar, A., Mansi, and David, M, Reeb. (2002). CEO Ownership and Managerial Incentives: Evidence from the Bond Market. Working paper, University of Alabama.
- Anderson, Ronald W. and Suresh Sundaresan. (1996). Design and Valuation of Debt Contracts. The Review of Financial Studies 9: 37-68.
- Aumann, R. and M. Perles. (1965). A Variational Problem Arising in Economics. Journal of Mathematical Analysis and Applications 11: 488-503.
- Bagnani, Elizabeth, S., Nikolaos, T, Milonas, Anthony, Saunders, and Nickolaos, G., Travlos. (1994). Managers, Owners, and the Pricing of Risky Debt: An Empirical Analysis. *Journal* of Finance 49: 453-478.
- Baker, Malcolm, Richard Ruback, and Jeffrey Wurgler. (2005). Behavioral Corporate Finance. The Handbook of Corporate Finance: Empirical Corporate Finance 145-188.
- Bebchuk, Lucian, and Jesse Fried, (2004). Pay Without Performance: The Unfulfilled Promise of Executive Compensation. Cambridge, MA: Harvard University Press.
- Bebchuk, Lucian, and Robert J. Jackson Jr. (2005). Executive pensions. Journal of Corporation Law 30, 823-855.
- Bekaert, Geert and Guojun Wu. (2000). Asymmetric Volatility and Risk in Equity Markets. The Review of Financial Studies 13: 1-42.
- Berk, Jonathan B., Richard Stanton, and Josef Zechner. (2006). Human Capital, Bankruptcy and Capital Structure. UC Berkeley Working Paper.
- Berndt, Antje, Rohan Douglas, Darrell Duffie, Mark Ferguson, and David Schranz. (2005). Measuring Default Risk Premia from Default Swap Rates and EDFs. Stanford University Working Paper.
- Berndt, Antje, Aziz A. Lookman, and Iulian Obreja. (2006). Default Risk Premia and Asset Returns. Carnegie Mellon University Working Paper.
- Bertrand, Marianne, and Sendhil, Mullainathan. (2003). Enjoying the Quiet Life? Corporate Governance and Managerial Preferences. *Journal of Political Economy* 111: 1043-q075.
 - Enjoying the Quiet Life? Corporate Governance and Managerial Preferences

- Brander, James, A., Michel, Poitevin. (1992). Managerial Compensation and the Agency Costs of Debt, *Managerial and Decision Economics* 13: 55-64.
- Cadenillas, Abel, Jakša Cvitanić, and Fernando Zapatero. (2004). Leverage Decision and Manager Compensation with Choice of Effort and Volatility. *Journal of Financial Economics* 73: 71-92.
- Campbell, John Y., and Glen B., Taksler. (2003). Equity Volatility and Corporate Bond Yields. Journal of Finance 58: 2321-2350.
- Carpenter, Jennifer N. (2000). Does Option Compensation Increase Managerial Risk Appetite? Journal of Finance 55: 2311-2331.
- Chen, Long, David A. Lesmond, and Jason Wei. (2007). Corporate Yield Spreads and Bond Liquidity. *The Journal of Finance* 62: 119-149.
- Clayton, Matthew C., Jay C., Hartzell, and Joshua, Rosenberg. (2005). The impact of CEO turnover on equity volatility. *Journal of Business* 78: 1779-1808.
- Coles, Jeffrey L., Naveen D. Daniel, and Lalitha Naveen. (2006). Managerial Incentives and Risk-Taking. Journal of Financial Economics 79: 431-468.
- Collin-Dufresne, Pierre and Robert S. Goldstein. (2001). Do Credit Spreads Reflect Stationary Leverage Ratios? Journal of Finance 56: 1929-1957.
- Collin-Dufresne, Pierre, Robert S. Goldstein, and J. Spencer Martin. (2001). The Determinants of Credit Spread Changes. *The Journal of Finance* 56: 2177-2207.
- Core, John and Wayne Guay. (2002). Estimating the value of employee stock option portfolios and their sensitivities to price and volatility. *Journal of Accounting Research* 40: 613-630.
- Core, John, E., Robert, Holthausen, and David, F., Larcker. (1999).Corporate Governance, Chief Executive Officer Compensation, and Firm Performance. Journal of Financial Economics 51: 371-406.
- Core, John, E., and David, F., Larcker. (2002). Performance Consequences of Mandatory Increases in Executive Stock Ownership. *Journal of Financial Economics* 64: 317-340.
- Cox, John C. and Chi-fu Huang. (1989). Optimal Consumption and Portfolio Policies When Asset Prices Follow a Diffusion Process. *Journal of Economic Theory* 49: 33-83.
- Daniel, Navven, D., J., Spencer, Martin, and Lalitha Naveen. (2005). Financial Market Consequences of Equity-Based Pay. Working Paper.
- Daniel, Kent, and, Sheridan Titman. (1997). Evidence on the characteristics of crosssectional variation in stock returns. *Journal of Finance* 52, 1-33.
- David, Alexander. (2007). Inflation Uncertainty, Asset Valuations, and the Credit Spreads Puzzle. *Review of Financial Studies* Forthcoming.
- Delianedis, G. and R. Geske. (1998). Credit Risk and Risk Neutral Default Probabilities: Information About Rating Migrations and Defaults. UCLA Working Paper.

- Peter M. DeMarzo, Yuliy Sannikov. (2006). A Continuous-Time Agency Model of Optimal Contracting and Dynamic Capital Structure. *Journal of Finance* 61, 2681-2724.
- Duarte, Jefferson, Francis A. Longstaff, and Fan Yu. (2007). Risk and Return in Fixed Income Arbitrage: Nickels in front of a Steamroller? *Review of Financial Studies* 20: 769-811.
- Duffie, Darrell. (1999). Credit Swap Valuation. Financial Analysts Journal 55: 73-87.
- Ekeland, Ivar and Thomas Turnbull. (1983). Infinite-Dimensional Optimization and Convexity. Chicago: The University of Chicago Press.
- Elton, Edwin J., Martin J. Gruber, Deepak Agrawal, and Christopher Mann. (2001). Explaining the Rate Spread on Corporate Bonds. *Journal of Finance* 56: 247-277.
- Eom, Young Ho, Jean Helwege and Jing-Zhi Huang. (2004). Structural Models of Corporate Bond Pricing: An Empirical Analysis. *Review of Financial Studies* 17: 499-544.
- Ericsson Jan. (2000). Asset Substitution, Debt Pricing, Optimal Leverage and Optimal Maturity. *Finance* 21: 36-69.
- Fahlenbrach, Rüdiger. (2008). Shareholder Rights, Boards, and CEO Compensation. *Review* of *Finance* Forthcoming.
- Fama, Eugene F. and Kenneth R. French. (1997). Industry Costs of Equity. Journal of Financial Economics 43: 153-193.
- Fenn, George W., and Nellie, Liang. (2001). DCorporate payout policy and managerial stock incentives. Journal of Financial Economics 60: 45-72.
- Fischer, Edwin O., Robert Heinkel, and Josef Zechner. (1989). Dynamic Capital Structure Choice: Theory and Tests. Journal of Finance 44: 19-40.
- Francois, P. and E. Morellec. (2004). Capital Structure and Asset Prices: Some Effects of Bankruptcy Procedures. Journal of Business 77: 387-411.
- Gabaix, Xavier, and Augustin Landier. (2008). Why Has CEO Pay Increased So Much? The Quarterly Journal of Economics 123: 49-100.
- Garen, John E. (1994). Executive Compensation and Principal-Agent Theory. The Journal of Political Economy 102: 1175-1199.
- Gibbons, Robert and Kevin J. Murphy (1992). Optimal Incentive Contractss in the Presence of Career Concerns: Theory and Evidence. *Journal of Political Economy* 100, 468-505.
- Gillan, Stuart, Jay Hartzell, and Robert Parrino (2008). Explicit vs. Implicit Contracts: Evidence from CEO Employment Agreements. Forthcoming in *Journal of Finance*.
- Goldstein, Robert, Nengjiu Ju, and Hayne Leland. (2001). An EBIT-Based Model of Dynamic Capital Structure. *Journal of Business* 74: 483-512.
- Graham, John R. (2000). How Big Are the Tax Benefits of Debt? *Journal of Finance* 55: 1901-1941.
- Graham, John R. and Lilian, Mills (2008). Using Tax Return Data to Simulate Corporate Marginal Tax Rates. *Journal of Accounting and Economics* forthcoming.

- Gomper, Paul, Joy, Ishii, and Andrew, Metrick. (2003). Corporate Governance and Equity Prices. Quarterly Journal of Economics 118: 107-155.
- Green, Richard C. (1984). Investment Incentives, Debt, and Warrants. Journal of Financial Economics 13: 115-136.
- Guay, Wayne, R., (1999). The Sensitivity of CEO Wealth to Equity Risk: An Analysis of the Magnitude and Determinants. *Journal of Financial Economics* 53: 43-78.
- Hennessy, Christopher A. (2004). Tobin's Q, Debt Overhang, and Investment. Journal of Finance 59: 1717-1742.
- Hennessy, Christopher A. and Toni M. Whited. (2005). Debt Dynamics. Journal of Finance 60: 1129-1165.
- Hennessy, Christopher A. and Toni M. Whited. (2007). How Costly Is External Financing? Evidence from a Structural Estimation. *Journal of Finance* 62: 1705-1745.
- Huang, Jing-Zhi and Ming Huang. (2003). How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk: A New Calibration Approach. Penn State University Working Paper.
- Holmstrom, Bengt. (1982). Moral hazard in teams. Bell Journal of Economics 19: 324-340.
- Huber, P.J. (1973), Robust Regression: Asymptotics, Conjectures and Monte Carlo. Annuals of Statistics 1: 799-821.
- Jensen, Michael C. and William H. Meckling. (1976). Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure. Journal of Financial Economics 3: 305-360.
- John, Teresa A. and Kose John. (1993). Top-Management Compensation and Capital Structure. Journal of Finance 48: 949-974.
- Jones, Philip E., Scott P. Mason, and Eric Rosenfeld. (1984). Contingent Claims Analysis of Corporate Capital Structures: An Empirical Investigation. *Journal of Finance* 39: 611-625.
- Karatzas, Ioannis, John P. Lehoczky, and Steven E. Shreve. (1987). Optimal Portfolio and Consumption Decisions for a "Small Investor" on a Finite Horizon. SIAM Journal of Control and Optimization 25: 1557-1586.
- Kim, E. Han. (1978). A Mean-Variance Theory of Optimal Capital Structure and Corporate Debt Capacity. *Journal of Finance* 33: 45-63.
- Klock, Mark, Sattar, Mansi, and William, Maxwell (2005). Does Corporate Governance Matter to Bondholders? *Journal of Financial and Quantitative Analysis* 40: 693-719.
- Lambrecht, Bart and Stewart Myers. (2006). Debt and Managerial Rents in a Real-Options Model of the Firm. MIT Working Paper.
- Leland, Hayne E. (1994). Debt Value, Bond Covenants, and Optimal Capital Structure. Journal of Finance 49: 1213-1252.
- Leland, Hayne E. (1998). Agency Costs, Risk Management, and Capital Structure. Journal of Finance 53: 1213-1243.

- Leland, Hayne E. and Klaus Toft. (1996). Optimal Capital Structure, Endogenous Bankruptcy and the Term Structure of Credit Spreads. *Journal of Finance* 51, 987-1019.
- Lewellen, Katharina. (2006). Financing Decisions When Managers Are Risk Averse. *Journal* of Financial Economics 82: 551-589.
- Longstaff, Francis A., Sanjay Mithal, Eric Neis. (2005). Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit Default Swap Market. *Journal of Finance* 60: 2213-2253.
- Luenberger, David G. (1969). *Optimization by Vector Space Methods*. New York: John Wiley and Sons, Inc.
- Mauer, David C. and Alexander J. Triantis. (1994). Interactions of Corporate Financing and Investment Decisions: A Dynamic Framework. *Journal of Finance* 49: 1253-1277.
- Mella-Barral, Pierre and William Perraudin. (1997). Strategic Debt Service. Journal of Finance 52: 531-556.
- Mello, Antonio S. and John E. Parsons. (1992). Measuring the Agency Costs of Debt. Journal of Finance 47: 1887-1277.
- Mehran H. (1992). Executive incentive plans, corporate control, and capital structure. Journal of Financial and Quantitative Analysis 27: 539-560.
- Merton, Robert C. (1974). On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *Journal of Finance* 29: 449-469.
- Modigliani, Franco, and Merton H. Miller, (1963). Corporate income taxes and the cost of capital: A correction. *American Economic Review* 53: 433-443.
- Morellec, Erwan. (2004). Can Managerial Discretion Explain Observed Leverage Ratios? Review of Financial Studies 17: 257-294.
- Moulton, Brent (1986). Random Group Effects and the Precision of Regression Estimates. Journal of Econometrics 32: 385-397.
- Moyen, Nathalie. (2000). Dynamic Investment Decisions with a Tax Benefit and a Default Cost of Debt. University of Colorado Working Paper.
- Murphy, Kevin J. and Ján Zábojník. (2004). CEO Pay and Appointments: A Market-based explanation for recent trends. *American Economic Review Papers and Proceedings* 94(2): 192-196.
- Murphy, Kevin J. and Ján Zábojník. (2006). Managerial Capital and the Market for CEOs. Working paper, Queen's University.
- Norden, Lars and Martin Weber. (2004). Informational Efficiency of Credit Default Swap and Stock Markets: The Impact of Credit Rating Announcements. *Journal of Banking & Finance* Vol. 28, Issue 11, November 2004: 2813-2843.
- Ortiz-Molina, Hernan. (2006). Top-Management Incentives and the Pricing of Corporate Public Debt. Journal of Financial and Quantitative Analysis 41: 317-340

- Ortiz-Molina, Hernan. (2007). Executive Compensation and Capital Structure: The Effects of Convertible Debt and Straight Debt. *Journal of Accounting and Economics* 43: 69-93.
- Parrino, Robert, Allen M. Poteshman, and Michael S. Weisbach. (2005). Measuring Investment Distortions When Risk-Averse Managers Decide Whether to Undertake Risky Projects. *Financial Management* 34: 21-60.
- Parrino, Robert and Michael S. Weisbach. (1999). Measuring Investment Distortions Arising from Stockholder-Bondholder Conflicts. *Journal of Financial Economics* 53: 3-42.
- Rockafellar, R. Tyrrell. (1970). Convex Analysis. Princeton: Princeton University Press.
- Ross, Stephen A. (2004). Compensation, Incentives, and the Duality of Risk Aversion and Riskiness. *Journal of Finance* 59: 207-225.
- Shleifer, Andrei, and Robert W., Vishny. (1992). Liquidation Values and Debt Capacity: A Market Equilibrium Approach. The Journal of Finance 47: 1343-1366.
- Shumway, Tyler (2001). Forecasting Bankruptcy More Accurately: A Simple Hazard Model. The Journal of Business 74: 101-124.
- Smith, Clifford W. and Ross L. Watts (1982). Incentive and tax effects of U.S. executive compensation plans. Australian Journal of Management 7: 139157.
- Strebulaev, Ilya A. and Stephen M. Schaefer. (2004). Structural Models of Credit Risk are Useful: Evidence from Hedge Ratios on Corporate Bonds. London Business School Working Paper.
- Sundaram, Rangarajan K. and David L. Yermack, (2007). Pay Me Later: Inside Debt and Its Role in Managerial Compensation. *Journal of finance* 62: 1551-1588.
- Talmor, Eli, Robert Haugen, and Amir Barnea. (1985). The Value of the Tax Subsidy on Risky Debt. Journal of Business. 58: 181-202.
- Titman, Sheridan and Sergey Tsyplakov. (2003). A Dynamic Model of Optimal Capital Structure. University of Texas at Austin Working Paper.
- Zechner, Josef and Peter Swoboda. (1986). The Critical Implicit Tax Rate and Capital Structure. *Journal of Banking and Finance* 10: 327-341.
- Yu, Fan. (2006). How Profitable Is Capital Structure Arbitrage? Financial Analysts Journal 62: 47-62.

Table I Summary Statistics

Our sample of 608 US firms is required to have executive compensation and financial information from the Compustat, ExecuComp, and CRSP databases as well as CDS rates from the Markit Group database during the time period January 2001-December 2006. Annual CDS rates are the average of month-end CDS rates within each calendar year. Leverage ratios (L/(S+L)) are the most recently reported debt book value (Compustat data items 9+34) divided by debt book value plus CRSP year-end market capitalization. Equity standard deviation σ is calculated each year at the firm level from daily CRSP returns during the fourth quarter. The Sharpe ratio is the five-year moving average of the calendar year ratios of average monthly CRSP excess returns to standard deviations. Sales, return on assets (ROA), stock returns (r), and CEO salary are the ExecuComp data items SALES, ROA, TRS1YR, and SALARY. Book-to-Market is calculated from CRSP and Compustat using the procedure of Daniel and Titman (1997). Div Dummy is one (zero) if dividends per share (Compustat data item 26) in the prior year are non-zero. The tax rate τ is the simulated corporate tax rates before financing from the website of John Graham, with missing values filled in using the procedure of Graham and Mills (2008). The "G" variable is the Gompers, Ishii, and Metrick (2003) index of strength of takeover provisions from the website of Andrew Metrick. Collateral is the ratio of inventories and property, plant, and equipment to the book value of assets (Compustat data items (3+8)/6). CEO age, tenure, and non-CEO tenure (the number of year that a CEO spent as a firm's employe prior to becoming a CEO) are from ExecuComp or, if that data is missing, from the websites sec.edgar.gov and zoominfo.com. Stock holdings p_s are obtained by dividing stock owned by the CEO excluding options (ExecuComp variable SHROWN_EXCL_OPTS) by the number of shares outstanding (ExecuComp variable SHRSOUT). Option holdings q are the sum of exercisable and non exercisable options (ExecuComp variables OPT_UNEX_EXER_NUM and OPT_UNEX_UNEXER_NUM) divided by the number of shares outstanding. The effective stock ownership p is calculated at the CEO-firm-year level using the method of Core and Guay (2002). The option moneyness K/S is defined as the ratio of the average option strike, calculated by using the Core and Guay (2002) method, to the end of year stock price. The variable New CEO is a dummy variable equal to unity when the CEO is appointed for less than one calendar year.

				10^{th}		90^{th}
Variable	Ν	Mean	Std	%ile	Median	%ile
Δ	Firm	Characte	ristics			
1	• 1 11 111	Characte	1150105			
CDS rate (bp)	608	153	274	26	70	311
Leverage ratio $L/(S+L)$ (%)	608	31.6	20.8	8.0	26.8	62.4
Stock return r (%)	598	15.9	21.6	-3.9	12.8	38.7
Sharpe Ratio SR (monthly)	608	0.110	0.099	0.000	0.101	0.224
Equity Std Dev σ (%/year)	608	33.4	13.1	19.8	30.3	49.9
Sales (millions)	600	$11,\!083$	20,081	1,321	$4,\!970$	25,718
Market cap (millions)	608	$16,\!057$	$33,\!107$	$1,\!379$	6,093	$36,\!640$
ROA (%)	600	3.65	5.33	-0.97	3.32	9.85
Book-to-Market	587	0.6	0.5	0.2	0.5	1.2
Div Dummy	608	0.69	0.43	0	1	1
Tax rate τ (%)	608	32.0	5.8	24.5	35.0	35.5
G	568	9.8	2.4	7	10	13
Collateral (%)	587	40.8	24.4	4.2	41.1	73.7
В	. CEO	Characte	eristics			
Salary A (thousands)	608	931	375	601	900	1,269
Stock holdings p_s (%)	601	1.37	4.50	0.03	0.19	1.93
Option holdings q (%)	608	0.96	0.96	0.17	0.67	2.09
$p = p_s + \Delta q \ (\%)$	601	2.14	4.70	0.21	0.85	4.12
Cash-to-Stock $\frac{A}{nS}$	601	0.108	1.412	0.004	0.021	0.101
Age (vears)	608	56	6	49	57	63
Tenure (years)	606	19	11	5	18	34
Non-CEO tenure (years)	606	11	10	0	9	26
K/S	608	0.73	0.21	0.47	0.77	0.94
New CEO	608	0.08	0.16	0	0	0.25

Table II CDS rates and compensation terms.

Coefficients from pooled OLS regressions of annual CDS rates on firm and CEO characteristics for 608 firms during the time period 2001-2006. The dependent variable is the logarithm of the average of firms' monthend CDS rates within each calendar vear. Cash-to-stock, ln(A/(pS)) is the logarithm of the ratio of CEO Salary A (ExecuComp SALARY) to the product of the effective fractional stock holding p, defined following the method of Core and Guay (2002), and CRSP market capitalization S. Equity standard deviation σ is calculated each year at the firm level from daily CRSP returns during the fourth quarter. Leverage ratios L/(L+S) are debt book values (Compute data items 9+34) divided by the sum of the debt book value and the year-end CRSP market capitalization. The Sharpe ratio SR is the five-year moving average of the calendar year ratio of average monthly CRSP excess returns to standard deviation. The tax rate τ is the simulated corporate tax rates before financing from the website of John Graham, with missing values filled in using the procedure of Graham and Mills (2008). The variables $ln(A/(pS)), \sigma, L/(S+L), SR$ and τ are measured at the end of the calendar year preceding the year in which CDS rates are observed. Sales, return on assets (ROA), and stock returns (r) are the logarithm of the ExecuComp data items SALES, 1+ROA, and 1+TRS1YR. The logarithm of Book-to-Market ln(B/M) is calculated from CRSP and Computat using the procedure of Daniel and Titman (1997). Div Dummy is one (zero) if dividends per share (Compustat data item 26) in the prior year are non-zero. The variables $\ln(\text{Sales})$, $\ln(1+\text{ROA})$, $\ln(1+r)$, $\ln(B/M)$ and Div Dummy are lagged by one year relative to the date at which CEO Salary and stock holdings are measured. The variable New CEO is a dummy variable equal to unity when the CEO is appointed for less than one calendar year. The "G" variable is the Gompers, Ishii, and Metrick (2003) index of strength of takeover provisions from the website of Andrew Metrick. All regressions include an intercept and year dummies (not reported). Industry fixed effects (not reported) are based on four digit-SIC codes following Fama and French (1997). T-statistics are based on standard errors that adjusted for the clustering of observations at the CEO-firm level.

Variable	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
$\ln\left(\frac{A}{pS}\right)$	0.053^{a}		0.042^{a}	0.054^{a}	0.038^{b}	0.039^{b}	0.060^{a}
	(3.27)		(2.83)	(3.02)	(2.38)	(2.16)	(3.02)
σ		0.023^a (15.56)	0.023^a (15.25)				
$\left(\frac{L}{S+L}\right)$	1.288^{a}	1.199^{a}	1.141^{a}	1.369^{a}	1.764^{a}	1.802^{a}	
$\left(S+L\right)$	(8.22)	(8.23)	(8.16)	(8.09)	(11.12)	(10.70)	
SR	-0.211 (-1.15)	-0.462^a (-2.72)	-0.317^c (-1.87)	-0.231 (-1.22)	$0.037 \\ (0.21)$	-0.009 (-0.05)	-0.395^b (-2.15)
τ	-1.444^{a} (-5.27)	-1.040^{a} (-4.54)	-0.952^a (-4.18)	-1.211^{a} (-4.48)	-1.179^a (-4.71)	-0.986^a (-3.99)	-1.325^a (-4.57)
$\ln(\text{Sales})$	-0.187^a (-8.52)	-0.183^a (-9.60)	-0.184^{a} (-9.39)	-0.181^a (-7.74)	-0.211^{a} (-10.06)	-0.207^{a} (-9.10)	-0.189^a (-7.64)
$\ln(1 + ROA)$	-1.912^a (-6.23)	-1.138^a (-4.13)	-1.080^{a} (-3.87)	-1.926^a (-5.03)	-1.986^a (-6.63)	-2.043^{a} (-5.49)	-2.989^a (-7.66)
$\ln(1+r)$	$0.000 \\ (0.01)$	$\begin{array}{c} 0.049 \\ (1.32) \end{array}$	$0.038 \\ (1.02)$	-0.008 (-0.20)	$0.043 \\ (1.13)$	$\begin{array}{c} 0.035 \ (0.89) \end{array}$	$0.064 \\ (1.46)$
$\ln(\mathrm{B/M})$	0.146^a (4.40)	$\begin{array}{c} 0.179^{a} \\ (5.85) \end{array}$	0.164^a (5.45)	$\begin{array}{c} 0.130^{a} \\ (3.62) \end{array}$	0.158^a (4.88)	0.142^a (4.08)	0.295^a (7.57)
Div Dummy	-0.771^{a} (-14.68)	-0.545^{a} (-11.40)	-0.560^a (-11.45)	-0.784^{a} (-13.97)	-0.585^a (-10.74)	-0.586^{a} (-10.26)	-0.673^a (-10.27)
New CEO				-0.016 (-0.28)		$0.003 \\ (0.06)$	
G				-0.002 (-0.22)		-0.006 (-0.60)	
Ind FE	no	no	no	no	yes	yes	yes
$N R^2$	$\begin{array}{c} 2441 \\ 0.52 \end{array}$	$\begin{array}{c} 2543 \\ 0.61 \end{array}$	$\begin{array}{c} 2441 \\ 0.62 \end{array}$	$\begin{array}{c} 2305 \\ 0.53 \end{array}$	$\begin{array}{c} 2441 \\ 0.58 \end{array}$	$\begin{array}{c} 2305 \\ 0.59 \end{array}$	$2441 \\ 0.51$

Significant at 1% (a), 5% (b), and 10% (c) levels.

Table III Robustness

Coefficients from regressions of annual CDS rates on firm and CEO characteristics for 608 firms during the time period 2001-2006. Regression (i) uses the Fama-McBeth technique, regression (ii) uses a regression technique that is robust to outliers (SAS Proc robustreg), and regressions (iii)-(vii) are pooled OLS. The dependent variable in all regressions is the logarithm of the average of firms' month-end CDS rates within each calendar year. Cash-to-stock, ln(A/(pS)) is the logarithm of the ratio of CEO Salary A (ExecuComp SALARY) to the product of the effective fractional stock holding p, defined following the method of Core and Guay (2002), and CRSP market capitalization S. The variable $ln(A/((p_s+q)S))$ is an alternative proxy of cash-to-stock where the sum of fractional stock and option holding $(p_s + q)$ replaces the effective stock holding p. Fractional stock holding p_s is the stock owned by the CEO excluding options (ExecuComp variable SHROWN_EXCL_OPTS) divided by the number of shares outstanding (ExecuComp variable SHRSOUT). Fractional option holding q is the sum of exercisable and non-exercisable options (ExecuComp variables OPT_UNEX_EXER_NUM and OPT_UNEX_UNEXER_NUM) divided by the number of shares outstanding. The variable ln((A + bonus)/(pS)) is another alternative proxy for cash-to-stock where cash is calculated by adding to the CEO salary A the CEO bonus (ExecuComp variable BONUS). The variable $q/(p_s+q)$ is the ratio of fractional option holdings to the sum of fractional stock and option holdings. Option moneyness K/S is the ratio of the CEO's average option strike, calculated using the Core and Guay (2002) method, to the end of year stock price. CEO age, tenure, and non-CEO tenure (the number of year that a CEO spent as a firm's employe prior to becoming a CEO) are from ExecuComp or, if that data is missing, from the websites sec.edgar.gov and zoominfo.com. Leverage ratio L/(L+S) is debt book value (Compustat data items 9+34) divided by the sum of the debt book value and year-end CRSP market capitalization. The Sharpe ratio SR is the five-year moving average of the calendar year ratio of average monthly CRSP excess returns to standard deviation. The tax rate τ is the simulated corporate tax rate before financing from the website of John Graham, with missing values filled using the procedure of Graham and Mills (2008). The variables ln(A/(pS)), $ln(A/((p_s+q)S))$, ln((A+bonus)/(pS)), L/(S+L), SR and τ are measured at the end of the calendar year preceding the year in which CDS rates are observed. Sales, return on assets (ROA), and stock returns (r) are the logarithm of the ExecuComp data items SALES, 1+ROA, 1+TRS1YR. The logarithm of Book-to-Market ln(B/M) is calculated from CRSP and Compustat using the procedure of Daniel and Titman (1997). Div Dummy is one (zero) if dividends per share (Compustat data item 26) in the prior year are non-zero. The variables $\ln(\text{Sales})$, $\ln(1 + \text{ROA})$, $\ln(1 + r)$, $\ln(B/M)$ and Div Dummy are lagged by one year relative to the date at which CEO Salary and stock holding are measured. All regressions include an intercept and year dummies (not reported). T-statistics are based on standard errors and for regressions (iii)-(vii) standard errors are adjusted for the clustering of observations at the CEO-firm level.

Variable	(i) FM	(ii) Robust	(iii) Options	(iv) Bonus	(v) All	(vi) Old	(vii) Young
$\ln\left(\frac{A}{\pi^{S}}\right)$	0.047^{a}	0.047^{a}	-1		0.064^{a}	0.088^{a}	0.056^{a}
(<i>ps</i>)	(4.19)	(5.19)			(3.25)	(2.70)	(3.00)
		× ,				~ /	~ /
$\ln\left(\frac{A}{(p_s+q)S}\right)$			0.073^{a}				
			(3.77)				
(4 + bonus)							
$\ln\left(\frac{A+BOHUS}{pS}\right)$				0.034^{a}			
				(2.67)			
$\frac{q}{\frac{1}{1}}$			-0.146				
p_s+q			(-1.52)				
			,				
K/S			-0.214^{b}				
			(-2.18)				
Age					-0.006^{c}	0.014	-0.020^{a}
					(-1.74)	(1.56)	(-3.78)
T					0.0070	0.001	0.0149
Tenure					(1.79)	-0.001	(2.80)
					(1.73)	(-0.21)	(2.00)
Non-CEO tenure					-0.017^{a}	-0.010^{c}	-0.024^{a}
					(-4.23)	(-1.73)	(-4.51)
$\begin{pmatrix} L \end{pmatrix}$	1 0000	1 1700	1.0770	1 00 40	1.0059	0.0000	1 4970
$\left(\frac{L}{S+L}\right)$	1.202^{a}	1.170°	1.277°	$1.294^{\circ\circ}$	1.295	(2.866°)	1.437°
	(10.10)	(14.41)	(8.11)	(7.93)	(8.58)	(3.88)	(8.15)
SR	-0.548^{b}	-0.202	-0.401^{b}	-0.326^{c}	-0.191	-0.173	-0.177
	(-2.04)	(-1.45)	(-2.06)	(-1.75)	(-1.05)	(-0.60)	(-0.81)
	1 5 7 40	1 9100	1 4070	1 4000	1 4010	1 09 <i>ch</i>	1 5000
au	$-1.5/4^{a}$	(-6.29)	-1.427^{α}	(-5.41)	(-5.26)	(-2.48)	(-4.59)
	(-0.01)	(-0.25)	(-0.12)	(-0.41)	(-0.20)	(-2.40)	(-1.00)
$\ln(\text{Sales})$	-0.178^{a}	-0.217^{a}	-0.181^{a}	-0.195^{a}	-0.162^{a}	-0.226^{a}	-0.125^{a}
	(-7.175)	(-16.82)	(-8.05)	(-9.09)	(-7.28)	(-6.29)	(-4.79)
$\ln(1 + ROA)$	-1.928^{a}	-1.985^{a}	-1.958^{a}	-1 909 ^a	-1.912^{a}	-2.356^{a}	$-1 727^{a}$
m(1 + non)	(-3.112)	(-8.14)	(-6.53)	(-6.20)	(-6.30)	(-4.57)	(-4.96)
	· · · ·	. ,	· · · ·	· · · ·	· · · ·	~ /	× ,
$\ln(1+r)$	0.067	-0.032	-0.012	-0.003	0.008	0.007	0.015
	(0.674)	(-0.80)	(-0.32)	(-0.08)	(0.20)	(0.10)	(0.34)
$\ln(B/M)$	0.139^{a}	0.157^{a}	0.136^{a}	0.154^{a}	0.140^{a}	0.145^{b}	0.141^{a}
	(4.534)	(7.16)	(4.05)	(4.59)	(4.16)	(2.45)	(3.73)
D. D	0			-		0.0-40	
Div Dummy	-0.737^{a}	-0.799^{a}	-0.745^{a}	-0.763^{a}	-0.705^{a}	-0.674^{a}	-0.687^{a}
	(-10.109)	(-23.09)	(-14.11)	(-14.00)	(-13.30)	(-1.11)	(-11.94)
N	407	2441	2441	2443	2428	741	1687
R^2	0.49	0.46	0.53	0.52	0.54	0.57	0.54

Significant at 1% (a), 5% (b), and 10% (c) levels.

Table IV Leverage Choice

Coefficients from pooled OLS regressions of leverage ratio on firm and CEO characteristics for 608 firms during the time period 2001-2006. The dependent variable is the leverage ratio L/(L+S) calculated by dividing the debt book value (Compute data items 9+34) by the sum of the debt book value and the year-end CRSP market capitalization. Cash-to-stock, A/(pS) is the logarithm of the ratio of CEO Salary A (ExecuComp SALARY) to the product of the effective fractional stock holding p, defined following the method of Core and Guay (2002), and CRSP market capitalization S. The Sharpe ratio SR is the five-year moving average of the calendar year ratio of average monthly CRSP excess returns to standard deviation. The tax rate τ is the simulated corporate tax rates before financing from the website of John Graham, with missing values filled in using the procedure of Graham and Mills (2008). The variables ln(A/(pS)), $ln(A/(pS))^2$ SR and τ are measured in the same year that leverage ratio is measured. Sales and return on assets (ROA) are the the ExecuComp data items SALES and 1+ROA. The logarithm of Book-to-Market ln(B/M) is calculated from CRSP and Compustat using the procedure of Daniel and Titman (1997). Collateral is the ratio of inventories and property, plant, and equipment to the book value of assets (Compustat data items (3+8)/6). Div Dummy is one (zero) if dividends per share (Compustat data item 26) in the prior year are non-zero. The variables $\ln(\text{Sales}), \ln(1 + \text{ROA}), \ln(B/M)$, Collateral and Div Dummy are lagged by one year relative to the date at which leverage is measured. All regressions include an intercept and year dummies (not reported). Industry fixed effects (not reported) are based on four digit-SIC codes following Fama and French (1997). T-statistics are based on standard errors that adjusted for the clustering of observations at the CEO-firm level.

Variable	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
$\ln(A/(pS))$	0.013^{b}	0.016^{a}	0.010^{b}	0.012^{a}	0.036^{a}	0.037^{a}	0.030^{a}	0.028^{a}
	(2.38)	(3.65)	(1.96)	(3.06)	(3.71)	(4.53)	(3.23)	(3.51)
$\ln(A/(pS))^2$					0.001^{a}	0.001^{a}	0.001^{a}	0.001^{a}
					(3.73)	(3.78)	(3.36)	(2.98)
SR			-0.177^{a}	-0.258^{a}			-0.154^{a}	-0.239^{a}
			(-3.70)	(-6.15)			(-3.29)	(-5.72)
au	-0.081	-0.121^{b}	-0.058	-0.090^{c}	-0.063	-0.111^{b}	-0.045	-0.084
	(-1.26)	(-2.17)	(-0.92)	(-1.67)	(-1.03)	(-2.03)	(-0.75)	(-1.60)
$\ln(\text{Sales})$	0.013^{b}	0.014^{b}	0.011^{c}	0.011^{c}	0.016^{a}	0.016^{a}	0.014^{b}	0.012^{b}
	(2.07)	(2.21)	(1.72)	(1.69)	(2.59)	(2.58)	(2.19)	(1.99)
$\ln(1 + ROA)$	-0.625^{a}	-0.603^{a}	-0.593^{a}	-0.552^{a}	-0.610^{a}	-0.583^{a}	-0.584^{a}	-0.540^{a}
. ,	(-5.25)	(-6.10)	(-5.02)	(-5.81)	(-5.14)	(-5.94)	(-4.94)	(-5.70)
$\ln(B/M)$	0.116^{a}	0.083^{a}	0.114^{a}	0.078^{a}	0.111^{a}	0.079^{a}	0.109^{a}	0.075^{a}
	(10.83)	(8.64)	(10.61)	(8.12)	(10.17)	(8.18)	(10.01)	(7.77)
Collateral	-0.017	0.051	-0.013	0.060	-0.027	0.053	-0.022	0.060
	(-0.59)	(1.34)	(-0.44)	(1.56)	(-0.96)	(1.38)	(-0.78)	(1.57)
Div Dummy	-0.005	-0.048^{a}	-0.005	-0.049^{a}	-0.009	-0.050^{a}	-0.008	-0.0514
	(-0.29)	(-3.12)	(-0.28)	(-3.24)	(-0.55)	(-3.24)	(-0.52)	(-3.32)
Ind FE	n	У	n	У	n	У	n	У
N	2384	2384	2384	2384	2384	2384	2384	2384
R^2	0.36	0.47	0.37	0.49	0.37	0.48	0.37	0.49

Significant at 1% (a), 5% (b), and 10% (c) levels.



Figure 1: Timeline describing the manager's decision problems.



Figure 2: The manager's optimal terminal wealth as a function of the pricing kernel ξ_T . The solid line represents the manager's terminal compensation from cash and stock of a levered firm in each terminal state as summarized by ξ_T . The dashed line represents the manager's terminal compensation if given an equally valued initial stake in the stock of an unlevered firm and no cash compensation. The figure is generated using the parameters r = 0.05, $\tau = 0.3$, $\alpha = 0.33$, A = 0.001, p = 0.01, L = 1.0, $\gamma = 1.1$, and T = 5. To set the initial stock price $S_0 = 1.0$ we choose $V_0 = 2.2$.



Figure 3: Volatility choice as a function of the present value of LPP and the cash-to-stock ratio $A/(pS_0^R)$. The solid and dashed lines show the LPP volatility at date t = 0.25 as a function of the present value of LPP for a manager compensated with cash and stock in a levered firm. The dash-dotted line gives the LPP volatility of an unlevered firm run by a manager with no cash compensation. The figure is generated using the parameters r = 0.05, $\tau = 0.3$, $\alpha = 0.33$, A = 0.0002 (solid line), A = 0.001 (dashed line), p = 0.01, L = 1.0, $\gamma = 1.1$, and T = 5. To set the initial stock price $S_0 = 1.0$ we choose $V_0 = 2.4$ (solid line) and $V_0 = 2.2$ (dashed line).



Figure 4: Credit spreads and stock volatility as a function of the leverage ratio and the cash-to-stock ratio $A/(pS_0^R)$. In Panel A the solid (dashed) line represents the initial credit spreads for debt of firms run by a managers compensated with cash-tostock $A/(pS_0^R) = 0.02$ ($A/(pS_0^R) = 0.01$) at various leverage ratios. The dash-dotted line represents the credit spreads of firms with LPP volatility ν^M that is constant over the firm's life. The parameters ν^M are chosen to match the initial instantaneous stock volatility of a firm with identical leverage that is run according to the R model with $A/(pS_0^R) = 0.02$. Panel B shows equity volatilities as a function of the stock-debt ratio. The figure is generated using the parameters r = 0.05, $\tau = 0.3$, $\alpha = 0.33$, A = 0.0002 (solid line), A = 0.001 (dashed line), p = 0.01, L = 1.0, $\gamma = 1.1$, and T = 5. Levels of V_0 are chosen to set the initial stock price $S_0 = 1.0$.



Figure 5: Hedge ratios as a function of the leverage ratio. The solid and dashed lines give the number of units of stock required to hedge the return on a bond issued by a firm run by a manager compensated with cash and stock. The dotted and dash-dotted lines give the equivalent hedge ratios for a firm with constant LPP volatility. The figure is generated using the parameters r = 0.05, $\tau = 0.3$, $\alpha = 0.33$, A = 0.0002 (solid line), A = 0.001(dashed line), p = 0.01, L = 1.0, $\gamma = 1.1$, and T = 5. To set the initial stock price $S_0 = 1.0$ we choose $V_0 = 2.4$ (solid line) and $V_0 = 2.2$ (dashed line).



Figure 6: The optimal choice of leverage (top panel) and the credit spread at issuance (bottom panel) as functions of the initial cash-to-stock ratio $A/(pS_0)$. The top panel shows how optimal leverage choice depends on the manager's compensation terms as summarized by the cash-to-stock ratio $A/(pS_0^R)$. The bottom panel shows how at-issue credit spreads depend on compensation terms. The figure is generated using the parameters r = 0.05, $\tau = 0.3$, $\alpha = 0.33$, $V_0 = 1$, $\gamma = 1.1$, and T = 5.



Figure 7: CDS rates vs leverage in the sample (log-linear scale). Each datapoint represents an observation of a leverage/CDS rate pair from our sample of 608 US firms during the period 2001-2006. Details on data sources and construction are in Table I.

Technical Appendix to Leverage Choice and Credit Spreads when Managers Risk Shift.

Outline

This Technical Appendix includes material of a technical nature relevant to the "Leverage Choice and Credit Spreads when Managers Risk Shift" and is organized as follows. Section B1 generalizes the R model to permit firm's LPP to have an exposure to idiosyncratic risk. Section B2 provides the pricing formulas for a version of the Merton (1974) model with taxes and bankruptcy cost. Section B3 presents an homogeneity property of the first stage model. Section B4 provides some geometric intuition supporting the results R1 and R2 from the proof of Proposition 1.

B1. Incorporating Idiosyncratic Risk

In this appendix we describe how our model can be modified to incorporate a non-systematic risk in the unlevered firm value V.

Assume that the firm cash payment has a systematic risk component (represented by the Brownian process z) as well as an idiosyncratic risk component driven by an independent Brownian process z^i . Assume also that the manager has the ability to control the total risk ν of the cash payment so that the resulting process for V becomes

$$\frac{dV_t}{V_t} = (r + \alpha \rho \nu_t)dt + \nu_t \left(\rho dz_t + \sqrt{1 - \rho^2} dz_t^i\right),\tag{45}$$

where $\rho \in [-1, 1]$ represents the exposure to market risk. Notice that increases and decreases in ν_t have the same effect on both the systematic and idiosyncratic sources of risk z and z^i and, therefore, the manager does not have the ability to control either source of risk in isolation. The case $\rho = 1$ nests the benchmark model (2). Further, define a new Brownian motion

$$\hat{z}_t = \rho z_t + \sqrt{1 - \rho^2} z_t^i$$

and the exponential martingale process

$$\frac{dN_t}{N_t} = -rdt - \alpha \rho d\hat{z}_t, \quad N_0 = 1.$$
(46)

The following lemma establishes that the exponential martingale N is the projection of the state price density ξ on the Gaussian subspace generated by the Brownian process \hat{z} .

Lemma 9 Let $\hat{\mathcal{F}}_t = \sigma(\hat{z}_s, s \leq t)$ denote the σ -algebra generated by the observation of the path $(\hat{z}_s)_{s \leq t}$. Then

$$N_t = E\left[\xi_t \mid \hat{\mathcal{F}}_t\right]$$

for all $0 \leq t \leq T$.

Proof. We define the new Brownian motion \hat{z}^* by

$$\hat{z}_t^* = -\sqrt{1-\rho^2}z_t + \rho z_t^i.$$

Note that, by construction, \hat{z}^* is independent from \hat{z} . Furthermore,

$$z_t = \rho \hat{z}_t + \sqrt{1 - \rho^2} z_t^*, \ z_t^i = \sqrt{1 - \rho^2} \hat{z}_t - \rho z_t^*.$$

Now,

$$\begin{split} E\left[\xi_t \mid \hat{\mathcal{F}}_t\right] &= E\left[e^{-rT}e^{-\frac{\alpha^2}{2}T - \alpha z_t} \mid \hat{\mathcal{F}}_t\right] \\ &= E\left[e^{-rt}e^{-\frac{\alpha^2}{2}t - \alpha\left(\rho\hat{z}_t + \sqrt{1 - \rho^2}z_t^*\right)} \mid \hat{\mathcal{F}}_t\right] \\ &= e^{-rt}e^{-\frac{\alpha^2}{2}t - \alpha\rho\hat{z}_t}E\left[e^{-\alpha\sqrt{1 - \rho^2}z_t^*} \mid \hat{\mathcal{F}}_t\right] \\ &= e^{-rt}e^{-\frac{\alpha^2}{2}t - \alpha\rho\hat{z}_t}E\left[e^{-\alpha\sqrt{1 - \rho^2}z_t^*}\right] \\ &= e^{-rt}e^{-\frac{\alpha^2}{2}t - \alpha\rho\hat{z}_t}e\frac{\alpha^2(1 - \rho^2)}{2}t \\ &= e^{-rt}e^{-\frac{\alpha^2\rho^2}{2}t - \alpha\rho\hat{z}_t} = N_t, \end{split}$$

where the fourth equality follows from the fact that \hat{z}^* is independent from \hat{z} .

One implication of the above lemma is that any payoff which is exclusively determined by the paths of \hat{z} (i.e. measurable with respect to $\hat{\mathcal{F}}_t$ for a given t > 0) can be priced either using the state price density ξ or its projection N. To see this, consider any $\hat{\mathcal{F}}_t$ measurable payoff χ . It follows from the law of iterated expectations and Lemma 9 that

$$E\left[\xi_{t}\chi\right] = E\left[E\left[\xi_{t}\chi \mid \hat{\mathcal{F}}_{t}\right]\right] = E\left[\chi E\left[\xi_{t} \mid \hat{\mathcal{F}}_{t}\right]\right] = E\left[N_{t}\chi\right].$$

The above result is critical for our extension because it says that the exponential martingale N is an *effective* state price density for all payoffs which are solely contingent on the paths \hat{z} . It turns out that under optimal behavior for the R model, the optimal LPP, V_T , depends only on the path of \hat{z} and we can consequently price any firm security by using N instead of the original state price density ξ . The following proposition substantiates this claim

Proposition 10 In presence of idiosyncratic risk, Given any $(L, p) \in (0, \infty) \times [0, 1]$, there exists a unique rational expectations equilibrium in which bondholders correctly anticipate the manager's risk choice ν . In this equilibrium the bankruptcy threshold V_b^R is consistent with (3) and the optimization problem (13) yields LPP value

$$V_T^R = \left[\overline{V} + \left(\overline{V} - V_b^R + \frac{A}{p(1-\tau)}\right) \left(\left(\frac{N_T}{\overline{N}}\right)^{-\frac{1}{\gamma}} - 1\right)\right] \mathbf{1}_{\{N_T \le \overline{N}\}},\tag{47}$$

where $\overline{N} \in (0, \infty)$ is the unique scalar such that $E(N_T V_T) = V_0$ and wher \overline{V} is defined by (15).

Proof. The proof of Proposition 10 is identical to the proof of Proposition 1. This is due to the fact that the *R* model problem under the state variables dynamic (1)-(2) is isomorphic to the *R* model with the state variable dynamic (45)-(46) with (α, ξ, z) substituted for $(\alpha \rho, N, \hat{z})$.

Therefore, all the pricing results for the benchmark model will also hold with a simple change in notation. For example, the bond price formula (19) becomes

$$B_t^R = Le^{-r(T-t)} \mathcal{N}(\hat{d}(t, 1, N_t/\overline{N}))$$

where the function \hat{d} is defined by

$$\hat{d}(t,x,n) = \left(-\ln n + r(T-t) - \frac{\alpha^2 \rho^2}{2} (1 - 2(1-x))(T-t)\right) / (\alpha \rho \sqrt{T-t}).$$

Similarly, the first stage problem can be solved using the same numerical procedures as for the benchmark model where the parameters are appropriately redefined.

B2. The M Model

In this appendix we derive valuation equations for a version of the Merton (1974) model, generalized to include taxes and bankruptcy costs (the M model).

In the M model the process V_t , defined in equation (2), has constant volatility ν . Stock and bond price dynamics from the M and R models can be made to converge at high equity values by setting $\nu = \nu^M \equiv \alpha/\gamma$, and we denote the resulting LPP process by V_t^M . In order to compute security prices that account for taxes and bankruptcy costs, the valuation formulas in Section 2 can be applied with $V_T = V_T^M$.

Bond prices are given by

$$B_t^M = Le^{-r(T-t)} \mathcal{N} \left(\gamma d(t, \tilde{\gamma}_1, V_b^M / V_t^M) \right) + V_t^M (1 - \delta_f) (1 - \tau) \mathcal{N} \left(-\gamma d(t, \tilde{\gamma}_2, V_b^M / V_t^M) \right),$$
(48)

where

$$\tilde{\gamma}_{1} = \frac{1}{2} \left(1 + \frac{1}{\gamma^{2}} \right), \ \tilde{\gamma}_{2} = 1 - \tilde{\gamma}_{1}, \ V_{t}^{M} = V_{0} \frac{e^{-\left(\frac{1-\gamma}{\gamma}r + \frac{\alpha^{2}}{\gamma^{2}}\right)t}}{\xi_{t}^{1/\gamma}},$$
(49)

and V_b^M solves equation (3) with $B_0 = B_0^M$. This is similar to the formula provided in Merton (1974) where the risky bond can be replicated by a long position in a riskless bond with face value L and a short put. In our context, two option contracts are required for the bond's replicating portfolio. The first is $(1 - \delta_f)(1 - \tau)$ units of a put on LPP, V_T^M , with strike price V_b^M . Due to the discontinuity in free cashflow at V_b^M a binary option is also required, paying the foregone tax shield $\tau(L - B_0^M)$ and the lump sum $\delta_f(1 - \tau)V_b^M$ when the firm is insolvent. The net payoff from shorting these options accounts for the fact that bondholders are residual claimants to non-zero firm value in the insolvent states, but themselves incur the costs of bankruptcy. Of course, they anticipate this payoff structure, as reflected in the pricing function (48). Yields in the M model are given by

$$y_t^M = r - \frac{\ln\left(\mathcal{N}\left(\gamma d(t, \tilde{\gamma}_1, V_b^M / V_t^M)\right) + \frac{(1-\tau)(1-\delta_f)V_t^M}{Le^{-r(T-t)}}\mathcal{N}\left(-\gamma d(t, \tilde{\gamma}_2, V_b^M / V_t^M)\right)\right)}{T-t}$$
(50)

and credit spreads are given by $\rho_t^M = y_t^M - r$. Bond return volatility is given by

$$\eta_t^M = \frac{\alpha}{\gamma} \frac{V_t^M}{B_t^M} (1 - \delta_f) (1 - \tau) \mathcal{N} \left(-\gamma d(t, \tilde{\gamma}_2, V_b^M / V_t^M) \right).$$
(51)

Stock payouts can be replicated by $(1 - \tau)$ units of a call option on the LPP with strike price V_b^M . The equity price is given by

$$S_t^M = (1-\tau) \left[V_t^M \mathcal{N} \left(\gamma d(t, \tilde{\gamma}_1, V_b^M / V_t^M) \right) - V_b^M e^{-r(T-t)} \mathcal{N} \left(\gamma d(t, \tilde{\gamma}_2, V_b^M / V_t^M) \right) \right]$$
(52)

and equity volatility is

$$\sigma_t^M = (1 - \tau) \frac{\alpha}{\gamma} \frac{V_t^M}{S_t^M} \mathcal{N}\left(\gamma d(t, \tilde{\gamma}_1, V_b / V_t^M)\right).$$
(53)

B3. Homogeneity Property of the First Stage Model

In this section, the model parameters $(\tau, \gamma, T, \alpha, r)$ are fixed. We will show that the first stage model inherits an homogeneity property from the R model described in Lemma 6. As a result, we prove (see Proposition 12) that the leverage ratio can be expressed as a function of cash-to-stock immediately after the optimal amount of debt is issued. Using numerical approximations, we depict the relationship between optimal leverage and cash-to-stock in Figure 6.

Recalling the R model's parameters (A, p, L, V_0) , we denote by $\mathcal{J}(A, p, L, V_0)$ the manager's expected utility (the value function) under optimal behavior for a firm run as in the R model. Using the explicit form of optimal LPP in Proposition 1, it can be shown that for any x > 0 and $(p, p') \in (0, 1)^2$,

$$\mathcal{J}\left(\frac{A}{px}, p', \frac{L}{x}, \frac{V_0}{x}\right) = \left(\frac{1}{px}\right)^{1-\gamma} \mathcal{J}(A, pp', L, V_0).$$
(54)

In the first stage problem, the firm/manager's exogenous parameters are now (A, p_0, V_0) . Recalling that \mathcal{J} is the value function for the R model, the managers first stage problem is to maximize

$$\mathcal{W}(A, p_0, V_0) = \sup_{L, p} \mathcal{J}(A, p, L, V_0)$$
(55)
s.t. $p_0(S_0^R + B_0^R) = pS_0^R,$
and $p \le 1,$

where (S_0^R, B_0^R) are the stock and bond price for a firm with parameters (A, p, L, V_0) when the manager behave according to the R model. To make the dependency on the model's parameters explicit, we will use the notation $(S_0^R, B_0^R) = \Xi(A, p, L, V_0)$. We also denote the solution of the optimization problem (55) by $(L(A, p_0, V_0), p(A, p_0, V_0))$.

Proposition 11 Consider a firm with exogenous parameters (A, p_0, V_0) and assume that the associated first stage model admits an interior solution $(L(A, p_0, V_0), p(A, p_0, V_0))$. For any x > 0, we have

$$L\left(\frac{A}{x}, p_0, \frac{V_0}{x}\right) = \frac{1}{x}L(A, p_0, V_0), \quad p\left(\frac{A}{x}, p_0, \frac{V_0}{x}\right) = p(A, p_0, V_0).$$
(56)

Furthermore, for any $p'_0 < 1$ and x > 0 such that the first stage problem with parameters $\left(\frac{A}{x}\frac{p'_0}{p_0}, p'_0, \frac{V_0}{x}\right)$ admits an interior solution, we have

$$L\left(\frac{A}{x}\frac{p_0'}{p_0}, p_0', \frac{V_0}{x}\right) = \frac{1}{x}L(A, p_0, V_0), \quad p\left(\frac{A}{x}\frac{p_0'}{p_0}, p_0', \frac{V_0}{x}\right) = \frac{p_0'}{p_0}p(A, p_0, V_0).$$
(57)

Proof. We will show simultaneously equation (56) and (57) by considering a firm with parameters $\left(\frac{A}{x}\frac{p'_0}{p_0}, p'_0, \frac{V_0}{x}\right)$. The first stage problem of the firm under consideration is

$$\mathcal{W}\left(\frac{A}{x}\frac{p'_{0}}{p_{0}}, p'_{0}, \frac{V_{0}}{x}\right) = \sup_{\tilde{L}, \tilde{p}} \quad \mathcal{J}\left(\frac{A}{x}\frac{p'_{0}}{p_{0}}, \tilde{p}, \tilde{L}, \frac{V_{0}}{x}\right)$$
s.t. $p'_{0}(S_{0}^{R,x} + B_{0}^{R,x}) = \tilde{p}S_{0}^{R,x}$
(58)

where $(S_0^{R,x}, B_0^{R,x}) = \Xi\left(\frac{A}{x} \frac{p'_0}{p_0}, \tilde{p}, \tilde{L}, \frac{V_0}{x}\right)$. Notice that the constraint $\tilde{p} \leq 1$ has been ignored because we assume that the problem admits an interior solution.

Using the homogeneity property of the R model formulated in equation (37) gives $(xS_0^{R,x}, xB_0^{R,x}) = \Xi\left(A, \frac{p_0}{p'_0}\tilde{p}, x\tilde{L}, V_0\right)$. On the other hand, the constraint of the problem (58) can be rewritten as $p_0(xS_0^{R,x} + xB_0^{R,x}) = \frac{p_0}{p'_0}\tilde{p}xS_0^R$. Using the property (54), we see that the optimization problem may be rewritten

$$\mathcal{W}\left(\frac{A}{x}\frac{p'_{0}}{p_{0}}, p'_{0}, \frac{V_{0}}{x}\right) = \left(\frac{p'_{0}}{xp_{0}}\right)^{1-\gamma} \quad \sup_{\tilde{L},\tilde{p}} \quad \mathcal{J}\left(A, \frac{p_{0}}{p'_{0}}\tilde{p}, x\tilde{L}, V_{0}\right)$$
(59)
s.t. $p_{0}(xS_{0}^{R,x} + xB_{0}^{R,x}) = \frac{p_{0}}{p'_{0}}\tilde{p}xS_{0}^{R,x}$

where we recall that $(xS_0^{R,x}, xB_0^{R,x}) = \Xi\left(A, \frac{p_0}{p'_0}\tilde{p}, x\tilde{L}, V_0\right)$. Comparing the structure of problem (59) and the problem (55), we see that their solutions are related. Specifically, if the control \tilde{L} is replaced by $\frac{L}{x}$ and the control \tilde{p} is replaced by $\frac{p'_0}{p_0}p$ in the problem (59), then the problem (59) becomes identical to problem (55). This observation shows that $L\left(\frac{A}{x}\frac{p'_0}{p_0}, p'_0, \frac{V_0}{x}\right) = \frac{1}{x}L(A, p_0, V_0)$ and $p\left(\frac{A}{x}\frac{p'_0}{p_0}, p'_0, \frac{V_0}{x}\right) = \frac{p'_0}{p_0}p(A, p_0, V_0)$ which proves both equation (56) and equation (57).

Proposition 11 implies that optimal leverage is a function of the after issuance cash-tostock. The next proposition formalizes this relationship between endogenous variables in the first stage model. In order to shorten notation we use the symbol L (resp. p) instead of $L(A, p_0, V_0)$ (resp. $p(A, p_0, V_0)$) when no confusion can arise.

Proposition 12 In the context of the manager's first stage problem, the optimal leverage can be expressed as a function of the cash-to-stock ratio and the model's exogenous parameters, that is, there exists a function Θ such that

$$\frac{L}{S_0^R + L} = \Theta\left(\frac{A}{pS_0^R}, \alpha, \gamma, \tau, r, T\right).$$
(60)

Proof. Fixing p'_0 and choosing $x = V_0$ in (57) shows that the endogenous variables $(V_0/L, p/p_0)$ only depend on the variable $A/(p_0V_0)$. Using the notation $S_0^R = S_0^R(A, p, L, V_0)$, we can choose $x = V_0$ in equation (37) and observe that S_0^R/V_0 is a function of $A/(pV_0)$ and V_0/L . Noticing that $A/(pV_0) = A/(p_0V_0) * (p/p_0)^{-1}$ and that both p/p_0 and V_0/L depend only on $A/(p_0V_0)$ shows that

$$\frac{S_0^R}{V_0} = \Gamma_1 \left(\frac{A}{p_0 V_0}\right)$$

for some function Γ_1 .

Now, the after issuance cash-to-stock ratio may be rewritten as $A/(pS_0^R) = [A/(p_0V_0)] * [p_0/p] * [S_0/V_0]^{-1}$ and since each of the three terms of this product depends on only $A/(p_0V_0)$ we see that $A/(pS_0^R)$ only depends on $A/(p_0V_0)$. On the other hand the stock to debt ratio may be rewritten as $S_0^R/L = (S_0^R/V_0) * (V_0/L)$ and the two terms of this product only depend on $A/(p_0V_0)$. As a result, equation (60) holds.

B4. Geometry of the L-F Transform

To build intuition for the basic principles underlying the results R1 and R2 from the proof of proposition 1, it is useful to consider the geometry behind the L-F transform in one dimension. To this end, it may be helpful to refer to Panel A of Figure A.1 when working through the following logic. Begin by assuming that h is a differentiable and convex function and notice that in this case the L-F transform can be defined as

$$h^*(x^*) = -\inf_{x \in \mathbb{R}} (h(x) - xx^*).$$

This alternative definition of the L-F transform shows that the problem consists of minimizing the distance between the function h and a line with slope x^* crossing the origin. If this line is translated by $-h^*(x^*)$ it will touch the graph of h only at x. We can thus equivalently define the L-F transform in terms of finding the highest line with slope x^* that lies below the function h. The equation of this line is

$$y = xx^* - h^*(x^*).$$

To build intuition for the results R1 and R2, observe that the first order conditions for (31)



are $h'(x) = x^*$ and the resulting L-F transform is given by $h^*(x^*) = xx^* - h(x)$. Totally differentiating this last expression with respect to x^* shows that we also have the symmetric condition $h^{*'}(x^*) = x$. The L-F transform (31) is therefore characterized by

$$h^*(x^*) + h(x) = xx^*, \quad h'(x) = x^*, \quad h^{*'}(x^*) = x.$$
 (61)

This formula reveals a fundamental symmetry between a convex differentiable function and its L-F transform. In particular, we see that if h^* is the L-F transform of h, then h must be the L-F transform of h^* . To see this, consider Panel B of Figure A.1 where we wish to find the solution to equation (32) at the point x^* . We now know that at this point the tangent to h^* has slope x and, by the first equation in (61), intercept -h(x). Thus $h^{**} = h$ and the result R1 is obvious in this simple setting. Result R2 is directly implied by the relationship (61) since the supporting lines are just the tangents.

Panel C in Figure A.1 illustrates how a function is convexified by applying the L-F transform twice. Consider first the function g_1 . Visual inspection shows that given any slope, the highest line lying below it cannot touch the graph in the non convex region $[x_l, x_h]$. Therefore, g_1^* cannot capture the variation in g_1 within this region. In particular, the functions g_1 and g_2 and, for that matter, any function lying above g_3 in the region $[x_l, x_h]$ will have the same L-F transform g_3^* . Hence, the double L-F transforms of g_1, g_2 , and g_3 are the same and must equal g_3 , the convex envelope of all of these functions.