# Signaling in Tender Offer Games

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#### Abstract

We examine whether a bidder can use the terms of the tender offer to signal the post-takeover security benefits. As atomistic shareholders extract all the gains in security benefits, signaling equilibria are subject to a constraint that is absent from bilateral trade models. The buyer (bidder) must enjoy gains from trade that are excluded from bargaining (private benefits) but can nonetheless be shared in a manner which allows inference about the security benefits. Restricted bids and cash-equity offers do not satisfy the condition. But firm-level governance provisions, debt financing and toeholds are viable signals. Similarly, the takeover probability in Hirshleifer and Titman (1990) permits the bidder to signal her type by forgoing private benefits through failure. While these signaling devices entail efficiencies, the inclusion of derivatives in the offer terms implements the full information outcome.

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## 1 Introduction

Presumably the best-known friction plaguing the market for corporate control is the freerider problem (Grossman and Hart, 1980; Bradley, 1980): Small shareholders perceive their individual decision as being negligible for the tender offer outcome, and hence do not tender unless the offer price matches at least the post-takeover share value. As a result, they extract all the gains in share value, which in turn may deter potential bidders.

Another friction which has received less attention is asymmetric information. In principle, both the bidder and the target shareholders can possess relevant private information. Contrary to merger negotiations between two management teams, the information advantage in tender offers is likely to be one-sided. Dispersed target shareholders do not actively monitor the firm and seldom possess information which is not already impounded in the stock price. By comparison, the bidder typically has spent resources to identify the target and to devise post-takeover (restructuring) plans. To succeed, the bidder therefore has to credibly communicate that the offer price adequately compensates target shareholders. Otherwise, the offer will be rejected even though it may entail a takeover premium.

The potential interaction of information asymmetries and coordination failures makes tender offers distinct from the standard bilateral trade setting. One-sided asymmetric information does not affect the bilateral trade outcome when the informed party makes a take-it-or-leave-it offer. This is not true in tender offer games. The first-mover advantage fails to endow the bidder with the ability to appropriate (part of the) gains in share value. Due to their free-riding behavior, uninformed shareholders have the bargaining power, even though the informed party (bidder) moves first.

This paper explores how and when bidders can signal their private information about the post-takeover security benefits. To the best of our knowledge, this questions has not yet been systematically analyzed, though it has been shown that separating equilibria can be constructed in tender offer games (Hirshleifer and Titman, 1990; Chowdhry and Jegadeesh, 1994). Yet, these papers do not analyse general existence conditions. Moreover, the proposed equilibria require dispersed shareholders to randomize their tendering decisions in a "coordinated" way to produce specific takeover probabilities. The present paper identifies the principle lying beneath the existence of signaling equilibria, thereby encompassing both previous results and novel signaling devices.

The paper's main insight is twofold. On the one hand, when cash flow and voting rights cannot be separated, the bidder's only way to signal her type is to forgo or share her private benefits through e.g., the choice of (firm-level) corporate governance provisions. However, signaling by means of manipulating private benefits entails inefficiencies as some value-increasing bids do not (always) occur. On the other hand, separating cash flow and voting rights is itself a viable signal that does not lead to any distortions. Thus, including

derivatives in the offer terms implements the full information outcome, irrespective of the existence or nature of private benefits.

Our analysis begins with a standard tender offer game in which a value-increasing bidder has exogenous private benefits and private information about the post-takeover security benefits. In this setting an impossibility result obtains: as shareholders extract all the gains in security benefits, the bidder cannot reveal her type through the offer terms. Thus, neither restricted bids nor cash-equity offers are viable signals, which stands out against findings from bilateral merger models (e.g., Eckbo et al., 1990). The result uncovers a fundamental conflict between incentive-compatibility and free-riding. The incentive-compatibility constraints require that high-valued bidders who have an incentive to mimic low-valued types earn information rents. However, the shareholders' free-rider behavior precludes that these rents come from gains in security benefits, as they are fully appropriated by the target shareholders. Absent private benefits, the free-rider problem thus precludes an incentive-compatible revealing offer.<sup>1</sup>

But even if the takeover is associated with private benefits, they can only serve a signaling purpose if they are not exogenously given. The bidder must be able to manipulate them to allow clear inference about the post-takeover security benefits. Hirshleifer and Titman (1990)'s probabilistic separating equilibrium showcases this principle. In their equilibrium, target shareholders randomize their tendering decision in a manner that lets bids at lower prices fail with higher probability. The higher failure probability deters high-valued bidders from mimicking low-valued types. Crucially, the deterrence operates exclusively through the fear of forgoing private benefits in the event of failure. If the bidder lacks such benefits, or high-valued bidders have substantially lower private benefits, the separating equilibrium breaks down. That is, bidders can credibly signal low security benefits only if lower bids forgo more (expected) private benefits, and if the corresponding loss is larger for high-valued bidders.

A principal contribution of this paper is to identify this mechanism as a broad principle for the viability of signaling in tender offer games. Accordingly, signaling equilibria can be implemented through firm-level governance provisions, toeholds and debt-financing, even when tendering decisions are deterministic. These signaling devices are all means that allow the bidder to choose how much of the proceeds to divert or withhold from target shareholders (Grossman and Hart, 1980; Shleifer and Vishny, 1986; Müller and Panunzi, 2004). For instance, the bidder can signal low security benefits by adopting stricter governance provisions, which make it more difficult to engage in self-dealing. (In the other cases, a low-valued bidder chooses a smaller toehold or less debt-financing to signal her type.) While this reduces private benefits, it allows her to succeed at a lower

<sup>&</sup>lt;sup>1</sup>The result can also be cast in terms of signaling costs. A trustworthy signal must be costly for the bidder. For example, the bidder might *voluntarily* forgo gains in security benefits (e.g., by rationing the trade). Yet, the free-rider problem precludes this possibility by *forcing* the bidder to forgo these gains.

price. The use of corporate governance provisions as a signal has unexpected implications. Firm quality and corporate governance provisions are inversely related as higher-valued bidders choose weaker corporate governance. Moreover, since higher-valued bidders pay higher prices, badly governed acquirers pay higher bid premia. Yet, the high bid premia neither reflect overpaying nor wasteful empire-building but merely the fact that the bidder creates more value.

In these signaling equilibria, lower-valued bidders must forgo more private benefits, either through failure or through reducing their level of private benefit extraction. As a result, the equilibrium outcomes typically exhibit inefficiencies at the "bottom": lowvalued bidders are more prone to fail or do not even submit a bid. In particular, when takeover outcomes are deterministic, only bidders above a cut-off type make a bid in equilibrium, and a lower cut-off value implies more value-improving takeover activity.

A common feature of these equilibria is that bid restrictions, though either insufficient or redundant as a signal, promote takeover activity. This is because smaller transaction sizes mitigate the asymmetric information problem: With fewer traded shares, a bidder gains less (in total) from paying a price below the post-takeover share value. This reduces the incentives to mimic low-valued bidders, so that these types do not need to sacrifice as much private benefits to credibly reveal low security benefits. Thus, more restricted bids translate into smaller signaling costs.

The positive impact of bid restrictions on takeover activity could be taken further if control did not require a majority stake. This insight leads us to reformulate the asymmetric information problem in tender offer games: control transfers are impaired because control must be transferred along with *misvalued* cash flow rights. The appropriate solution is therefore to separate votes from cash flow rights. Indeed, we show that the use of non-voting shares or financial derivatives can generate signaling equilibria that completely eliminate the impact of asymmetric information. These financial instruments allow the bidder to buy the target shares against cash, strip the shares of their votes, repartition the cash flow rights and reissue only those cash flow rights that she wants to shed. While the first two steps give the bidder control, the last two steps can be used to penalize "lying" about the post-takeover security benefits. In particular, call options enable target shareholders to seek "damages" from the bidder ex post if the security benefits turn out to be higher than ex ante professed. This makes the bid price de facto contingent on the post-takeover security benefits, thereby overcoming the information asymmetry. When the value improvement is deterministic, the options are never exercised so that the bidder essentially succeeds with a simple cash offer.

The use of derivatives allows to implement the full information outcome because of a crucial difference between the tender offer game and most other signaling models in corporate finance. In tender offer games, the gains from trade are typically realized upon the transfer of control, as opposed to the transfer of cash flow rights. Thus, in the market for corporate control, company shares represent a bundle of two "goods", cash flow and voting rights. Separating these goods is beneficial when frictions in the trade of one impose a negative externality on the trade of the other.

Grossman and Hart (1981) and Shleifer and Vishny (1986) offer the first analyses of asymmetric information in tender offer games. Both papers focus exclusively on pooling equilibria. At et al. (2008) revisit the pooling equilibrium and show that dual-class share structures mitigate the asymmetric information problem. Hirshleifer and Titman (1990) and Chowdry and Jegadeesh (1994) study tender offer games in which takeover outcomes are probabilistic. As we demonstrate in this paper, their separating equilibria are applications of a general principle which does not rely on the probabilistic tender offer outcome.

Several papers show that the choice of payment method can overcome asymmetric information problems in mergers (Hansen, 1987; Fishman, 1989; Eckbo et al., 1990; Berkovitch and Naranayan, 1990). Importantly, all of these papers consider bilateral merger negotiations and hence abstract from the free-riding problem. With the exception of Berkovitch and Naranayan, these papers consider two-sided asymmetric information settings in which target shareholders know more either about the share value under the incumbent manager or the takeover synergies. Thus, the settings differ from ours in precisely those aspects that are characteristic of tender offers. The same holds true for Brusco et al. (2007) and Ferreira et al. (2008) who study cash-equity offers in a mechanism design framework. The problem they explore becomes rather simple under our informational assumptions, and pure cash offers would always implement the full information outcome.

The paper proceeds as follows. The next section presents the basic model with exogenously given private benefits. Section 3 shows that this model has no signaling equilibria unless the tender offer outcome is probabilistic, and explains the importance of the freerider problem for this result. In addition, we demonstrate that neither bid restrictions nor cash-equity offers are viable signals in this setting. Section 4 shows why and how signaling equilibria can be implemented through firm-level governance provisions, debt-financing or toeholds. It also shows that cash-equity offers involving bidder assets can *only* serve as signals if the value of these assets is perfectly correlated with the security benefits and appreciates after the takeover. Section 5 demonstrates how the full information outcome can be implemented through the use of derivatives as a means of payment. Concluding remarks are in Section 6, and mathematical proofs are in the Appendix.

# 2 The Model

Our basic setting closely follows existing tender offer models with asymmetric information (Shleifer and Vishny, 1986; Hirshleifer and Titman, 1990), while remaining agnostic about the specific source of bidder gains. There is a widely held firm that faces a single potential acquirer, henceforth the bidder. If the bidder gains control, she can generate security benefits X. The bidder learns her type prior to making the tender offer, whereas target shareholders merely know that X is distributed on  $\mathcal{X} = [0, \overline{X}]$  according to the continuously differentiable density function g(X). The cumulative distribution function is denoted by G(X). If the takeover does not materialize, the incumbent manager remains in control. The incumbent can generate security benefits which are known to all shareholders and normalized to zero. Thus, we restrict attention to the case of value-improving bids.

In addition, control confers exogenous private benefits  $\Phi \geq 0$  on the bidder. The private benefits are only known to the bidder and for simplicity a deterministic function of her type.<sup>2</sup> Furthermore, the bidder cannot commit not to extract the private benefits once she is in control. As the private benefits ultimately accrue exclusively to the bidder, they are de facto non-transferable. Our specification of private benefits gains can accommodate various sources of bidder gains, such as dilution (Grossman and Hart, 1980) and toeholds (Shleifer and Vishny, 1986). Though, for the sake of notational simplicity, we subsequently assume that the bidder has no initial stake.

As the firm has a one share - one vote structure, a successful tender offer must attract at least 50 percent of the firm's shares. The tender offer is conditional, and therefore becomes void if less than 50 percent of the shares are tendered. In addition, the bidder can restrict the offer to a fraction  $r \in [0.5, 1]$  of the shares. For simplicity, we assume that there are no takeover costs. Hence, the benchmark (full information) outcome is that all takeovers succeed.<sup>3</sup>

The timing of the model is as follows. In stage 0, the bidder learns her type X. In stage 1, she then decides whether to make a take-it-or-leave-it, conditional, restricted tender offer in cash. (Alternative means of payment will be considered later.) If she does not make a bid, the game moves immediately to stage 3. Otherwise, she offers to purchase a fraction r of the outstanding shares at a price rP.

In stage 2, the target shareholders non-cooperatively decide whether to tender their shares. Shareholders are homogeneous and atomistic. In other words, they do not perceive

<sup>&</sup>lt;sup>2</sup>We will point out when results are not robust to private benefits being a random variable, distributed on  $[0, \bar{\Phi}]$  according to some conditional density function h(.|X).

<sup>&</sup>lt;sup>3</sup>Like other tender offer models exploring the free-rider problem, we assume that the firm's outstanding shares of mass 1 are dispersed among an infinite number of shareholders whose individual holdings are both equal and indivisible. When either of these assumption is relaxed, the Grossman and Hart (1980) result that all the gains in security benefits go to the target shareholders no longer holds (Holmström and Nalebuff, 1992).

themselves as pivotal for the tender offer outcome. In stage 3, the incumbent manager remains in control if the fraction of tendered shares  $\beta$  is less than 50 percent. Otherwise, the bidder gains control and pays  $\beta P$  unless the offer is oversubscribed, in which case she pays rP, and tendering shareholders are randomly rationed.<sup>4</sup>

As we will show, the bidder's (in)ability to signal her type is sensitive to the chosen setup. We henceforth refer to the basic model as the tender offer game with exogenous private benefits. Finally, to select among multiple Perfect Bayesian Equilibria, we use the Pareto-dominance criterion and, on occasion, the credible beliefs criterion (Grossman and Perry, 1986).

## 3 Equilibrium Outcomes

In this game, only the bidder has private information, and she moves first. If target shareholders could freely coordinate, they would tender whenever the offered price at least matches the security benefits under the incumbent manager. Thus, their reservation price would be independent of the bidder's type. Any bidder matching this price would succeed and appropriate the entire value improvement from the takeover.

However, when the shareholders are non-pivotal and non-cooperative, their reservation price depends on the bidder's type. Each of them tenders at stage 2 only if the offered price at least matches the expected security benefits. Since shareholders condition their expectations on the offer terms (r, P), a successful tender offer must satisfy the free-rider condition  $P \ge E(X|r, P)$ . We assume that shareholders do not play weakly dominated strategies. This eliminates failure as an equilibrium outcome when the free-rider condition is strictly satisfied.<sup>5</sup>

When the bid price exactly equals the expected post-takeover share value, the target shareholders are strictly indifferent between tendering and retaining their shares. That is, they are indifferent between these actions irrespective of their beliefs about the takeover outcome, so that the weak dominance criterion does not pin down a tendering strategy. The prevalent way of resolving the indeterminacy of the aggregate outcome when P = E(X | r, P) is to assume that each shareholder tenders in this case, and hence

<sup>&</sup>lt;sup>4</sup>Under the Williams Act partial bids may not be made on a first come first served basis, and bidders are required to prorate tendering shareholders (DeMott, 1988). The requirement to treat all shareholders of the same class equally in the UK City Code and in the proposed EU Takeover Directive are rules to the same effect.

<sup>&</sup>lt;sup>5</sup>Given a bid is conditional, a shareholder who believes the bid to fail is indifferent between tendering and retaining. Imposing this belief on all shareholders and breaking the indifference in favour of retaining supports failure as an equilibrium, irrespective of the offered price (Burkart et al., 2006). To avoid coexistence of success and failure as equilibrium outcomes, it is typically assumed that shareholders tender when they are indifferent (e.g., Shleifer and Vishny, 1986). Contrary to our assumption, this precludes failure as the equilibrium outcome for a conditional bid, and hence the existence of an equilibrium when the free-rider condition is violated.

the bid succeeds with certainty.<sup>6</sup> Alternatively, one may assume that strictly indifferent shareholders randomize, in which case the aggregate outcome may be probabilistic.<sup>7</sup> For either case, we subsequently analyze whether the game has fully revealing equilibria, in which (some) bidders reveal their type through the chosen offer terms (r, P). In addition, pooling equilibria exist in the tender offer game with both deterministic and probabilistic outcomes. To keep focus on the feasibility of signaling, the discussion of the pooling equilibria is relegated to Appendix A.

## **3.1** Deterministic Outcomes

Under the assumption that each shareholder tenders in case she is strictly indifferent, all shares ( $\beta = 1$ ) are tendered in a successful takeover. Accordingly, a successful restricted bid is oversubscribed, and the bidder randomly selects the fraction r among all shareholders whose shares she purchases. The remaining 1 - r shareholders cannot sell and become minority shareholders.

The bidder's expected profit from a bid (r, P) is

$$\Pi(r, P) = q(r, P) \left[ \Phi(X) + r \left( X - P \right) \right]$$

where q(r, P) denotes the aggregate success probability which is equal to 1 for  $P \ge E(X|r, P)$  and 0 otherwise. In a fully revealing equilibrium, the offer terms must be distinct across types that make a (successful) bid. This requires that each equilibrium offer satisfies the free-rider condition,  $P(X) \ge X$ , and the bidder's incentive-compatibility constraint

$$\Phi(X) + r(X)[X - P(X)] \ge \Phi(X) + r(X - P)$$

for all  $r \in [0.5, 1]$  and  $P \in \mathbb{R}$ .

**Proposition 1** In the deterministic tender offer game with exogenous private benefits, no fully revealing equilibrium exists.

Given that  $P(X) \ge X$ , a truthful bidder at best breaks even on the purchased shares, and her expected profit cannot exceed  $\Phi(X)$ . However, each type offering (at least) her actual security benefits cannot be an equilibrium outcome. If a type x would succeed with an offer rx, any type X > x would mimic type x to acquire shares at a price below their true value X. This also holds if each type would choose a different bid restriction r(X). Type X's profits are higher when buying r(x) shares at a discount compared to

<sup>&</sup>lt;sup>6</sup>A common motivation for this approach is that the bidder could sway the shareholders by raising the price infinitesimally. Although this argument holds under full information, it does not apply in the asymmetric information setting, as even small price increases affect shareholders' expectations about the post-takeover security benefits.

<sup>&</sup>lt;sup>7</sup>Judd (1985) shows that a continuum of i.i.d. variables can generate a stochastic aggregate outcome.

buying r(X) shares at their fair price whether r(x) < r(X) or r(x) > r(X) holds. These arguments eliminate P(X) = X combined either with a common r or a type-contingent r(X) as possible equilibria. They also rule out outcomes in which some types offer more than their true security benefits but less than the highest type's security benefits. Successful offers with  $P(x) \in (x, \overline{X})$  would be mimicked by bidders of type X > P(x). Thus, a bidder can credibly signal her type only by offering a sufficiently large premium such that  $P \ge \overline{X}$ .

Revealing her type with an offer  $P \ge \overline{X}$  is, however, not an attractive option for the bidder.<sup>8</sup> She can instead make a bid  $P = \overline{X}$  and restrict it to r = 0.5, the minimum fraction required to gain control. The less costly offer  $(0.5, \overline{X})$  succeeds as it satisfies the free-rider condition for all types (and any possible shareholder beliefs).

It is worth mentioning that the inexistence of a separating equilibrium does not depend on the assumption that the private benefits are a deterministic function of the bidder's type. The result extends to settings where private benefits follow some - possibly type-contingent - density function. Indeed, the constraints of the bidder's maximization problem are not affected by the exogenous private gains. They cancel out in the bidder's incentive-compatibility constraint and they are not part of the free-rider condition.

Also, letting bidders choose the fraction of shares that they acquire does not enable them to signal their type. If the bidder purchases shares at their true value - as she would in a fully revealing equilibrium - her profits do not depend on the choice of r. That is, r is not a viable signal. Its sole function is to limit the fraction of shares the bidder purchases in exchange for cash. This makes restricted bids in this setting equivalent to bids in which target shareholders are in part compensated through equity. Indeed, it is immaterial whether the bidder makes a partial bid for cash only or acquires all shares in exchange for some cash and 1 - r shares in the target firm under her control. Moreover, control requires that the partial bid is for at least half the shares and that the equity component does not exceed the cash component in the cash-equity offer. By virtue of this equivalence, any fully revealing equilibrium in cash-equity offers would also have to exist in restricted cash only offers.

# **Corollary 1** Introducing cash-equity offers into the deterministic tender offer game with exogenous private benefits does not make fully revealing equilibria feasible.

Corollary 1 conflicts with results from means-of-payment models where cash-equity offers can reveal the bidder's type (Hansen, 1987; Berkovitch and Naranayan, 1990; Eckbo

<sup>&</sup>lt;sup>8</sup>In fact, there exists an incentive-compatible schedule  $\{(r(\cdot), P(\cdot))\}$  which entails that lower-valued bidders offer higher (per-share) prices but purchase fewer shares. Higher-valued bidders abstain from mimicking as they would forego a larger more valuable equity stake. Conversely, higher-valued bidders offer larger total amounts,  $r(\cdot)P(\cdot)$ , which are unattractive for lower-valued bidders who would at the same expense purchase a less valuable equity stake. This is the logic underlying the fully revealing equilibrium in Berkovitch and Naranayan (1990).

et al., 1990).<sup>9</sup> Our basic framework differs from these merger models in two key respects. First, target shareholders have no private information. Instead, they face a collective action problem, i.e., are unable to coordinate their individual tendering decisions.<sup>10</sup> Second, the takeover is not undertaken to combine assets from two firms but to replace the incumbent managers. How or whether the free-rider problem affects signaling equilibria is the subject matter of the next section, while the role of bidder assets will be explored later in the paper (section 4).

#### 3.1.1 Free-Riding and Information Rents

To illustrate the role of the free-rider condition, we consider a modified setting in which the bidder is able to appropriate part of the security benefits. Abstracting from a specific extensive form, we assume that the bidder has bargaining power  $\omega \in (0,1)$  such that shareholders, if fully informed, would tender at a price  $P = (1 - \omega)X$ . Accordingly, the bidder would under full information appropriate a value improvement  $\omega X$  on the purchased shares. Like the private benefits  $\Phi$ , these gains depend on the bidder type and a successful takeover. But unlike the private benefits, the gains are transferable. That is, the bidder can (commit to) leave part of  $\omega X$  to the shareholders. A second, purely simplifying modification is the absence of private benefits ( $\Phi = 0$ ).

Given shareholders do not observe the bidder's type, they condition their beliefs on the offer terms and tender only if  $P \ge (1 - \omega) \to (X | r, P)$ . Consequently, some bidder may not succeed or realize less than the full information profit  $\omega X$ . That is, some bidders may have to offer more than  $(1 - \omega) X$  or set r < 1 to signal their type, while others may find such signals too costly.

**Proposition 2** In the tender offer game with bidder bargaining power  $\omega \in (0, 1]$ , a fully revealing equilibrium exists. All types above the cut-off type  $X^c(\omega) \in [0, \overline{X})$  make a bid, and higher types buy more shares at a higher price and make a larger profit.

Incentive compatibility requires that both elements of the  $\{(r(\cdot), P(\cdot))\}$  schedule increase with the bidder type. Higher-valued bidders offer to buy more shares at higher prices. On the one hand, bidders refrain from mimicking lower-valued types because the

<sup>&</sup>lt;sup>9</sup>Contrary to negotiated mergers, tender offers are usually cash offers. In fact, the mode of acquisition is one of the most important determinants of the payment method (e.g., Martin, 1996). The standard explanation focuses on regulatory delays associated with equity offers, i.e., the greater cost of using equity as a means of payment. We suggest that, in addition, the benefit of using equity may be smaller in tender offers, which also makes the use of equity less attractive. Evidence that the signaling benefits of equity may indeed vary with the target's ownership structure, and the (implied) information distribution, comes from differences in the announcement-day returns of publicly and privately held targets (Chang, 1998).

<sup>&</sup>lt;sup>10</sup>In merger models, the shareholders' reservation price is typically the stand-alone value of the target firm that although unknown to the bidder does not depend on her type. A notable exception is Berkovitch and Narayanan (1990) where shareholders' outside option is to wait for a competing bid, and the option value depends on the quality of the initial bidder relative to potential competitors.

increase in the profit margin is offset by the smaller fraction of shares that can be bought at lower prices. On the other hand, bidders do not mimic higher-valued types as buying more shares requires paying a higher price.<sup>11</sup> Lower-valued bidders credibly reveal their type not only by bidding for fewer shares but also by purchaising them at lower relative discounts [P(X) - X]/X, thereby having to concede an increasing part of their full information profit  $\omega X$  to the target shareholders.<sup>12</sup> Thus, equilibrium profits are increasing in bidder type, and there exists a cut-off type  $X^c$  who just breaks even, offering  $P = X^c$ (and having no private benefits). Conversely, the highest-valued bidder can reap her full information gains  $\omega \overline{X}$ . She can purchase all shares at  $P = (1 - \omega) \overline{X}$  because target shareholders always tender for  $P \ge (1 - \omega) \overline{X}$  irrespective of their beliefs.<sup>13</sup>

Due to above equivalence result (Corollary 1), Proposition 2 can also be phrased in terms of cash-equity offers. In this interpretation, the equilibrium offer schedule entails that higher-valued bidders use more cash and less equity. This is the same result as in the bilateral merger models of Eckbo et al. (1990) and Berkovitch and Narayanan (1990), though there is a subtle difference. In our setting, the bidder wants to signal low rather than high security benefits. This shifts the emphasis from cash as a high-value signal to equity as a low-value signal. Equity is a credible signal for low-valued bidders because relinquishing equity is costlier for high-valued bidders.

The positive relation between profits and bidder types in Proposition 2 is a common feature of adverse selection models: Incentive-compatibility requires that types who have incentives to mimic others earn information rents (e.g., Laffont and Martimort, 2002). In our setting, higher-valued bidders receive these rents so that incentive-compatibility dictates a slope at which equilibrium profits increase. Given the slope, the profit levels are determined by the boundary condition  $\Pi^*(\overline{X}) = \omega \overline{X}$ . That is, the highest type's equilibrium profit determines at which type the incentive-compatible profit falls to 0.

# **Proposition 3** As the bidder's bargaining power $\omega$ approaches 0, the cut-off type $X^{c}(\omega)$ converges to $\overline{X}$ .

The proposition brings to light the impact that the free-riding behavior has on the feasibility of revealing offers. The subset of bidders that can signal their type without incurring a loss (on the purchased shares) monotonically decreases in the fraction of the share value improvement that each bidder could appropriate under full information.

<sup>&</sup>lt;sup>11</sup>Proposition 2 implies that the bidder faces an upward-sloping supply curve. A larger demand reveals a higher valuation which in turn raises the shareholders' ask price. This is akin to the downward sloping demand curve that a privately informed issuer meets when selling securities (e.g., DeMarzo and Duffie, 1999). Though contrary to the informed seller setting, gains from trade materialize in the tender offer game only if the bidder acquires a control stake. Therefore, trade collapses once the incentive-compatible supply of shares is less than 0.5.

<sup>&</sup>lt;sup>12</sup>Absent the control constraint  $r \ge 0, 5$ , the relative discount would increase at a lower rate, allowing more bidder types to purchase (fewer) shares at a profit.

<sup>&</sup>lt;sup>13</sup>The fully revealing equilibrium outcome is supported by shareholders' out-of-equilibrium beliefs that attribute any deviating offer to the highest-valued bidder.

In other words, as shareholders' free-riding behavior becomes more severe, the bidder's ability to signal her type gradually deteriorates. In the limit ( $\omega = 0$ ), no bidder type makes a profit on the purchased shares, and the separating equilibrium breaks down.

To be noted is that separation fails even though the bidder's objective function satisfies the single-crossing property.<sup>14</sup> The impact of the free-rider condition on the (in)existence of signaling equilibria can be interpreted in two ways. From the perspective of lowervalued types, the free-rider condition eliminates the possibility of producing a costly signal. Given that target shareholders extract all the gains in security benefits, the bidder cannot surrender (part of) these gains to signal her type. Taking the perspective of higher-valued types, the free-rider condition wipes out information rents. A bidder who at best breaks even on truthfully purchased shares always mimics a lower bid price, provided that her private benefits are independent of the offer terms.

When  $\omega = \Phi = 0$ , neither a fully revealing nor a pooling equilibrium exists (cf. Appendix A), and trade virtually collapses as only the highest type makes a bid. It is worth comparing this outcome to Milgrom and Stokey (1982)'s no-trade theorem. The theorem says that asymmetrically informed but rational parties cannot agree to a transaction unless there are aggregate gains to be shared. In the tender offer game, such gains are present, as the takeover improves the target's value. Still, the tender offer is from the bidder's perspective equivalent to a trade without any aggregate gains, as the latter are entirely appropriated by the free-riding shareholders. Thus, trade breaks down because, albeit the bidder makes a take-it-or-leave-it offer, shareholders have full bargaining power.

## 3.2 Probabilistic Outcomes

In this section, we assume that strictly indifferent shareholders randomize their tendering decisions, and that the randomized decisions can in aggregate produce a probabilistic tender offer outcome. More specifically, if P = E(X|r, P), the probability q(r, P) that the bid succeeds can therefore lie anywhere in the interval [0, 1], whereas the expected fraction of shares  $\gamma(r, P)$  acquired by a successful bidder can lie anywhere in the interval [0.5, r]. The response functions q(r, P) and  $\gamma(r, P)$  thus characterize how shareholders as a group react to an observed bid (r, P).

When the bid price differs from the post-takeover security benefits, tender offer outcomes are deterministic: shareholders always tender if offered more, and never tender if offered less. One can therefore invoke the same arguments that lead to Proposition 1 to rule out fully revealing equilibria with  $P(X) \neq X$  in the probabilistic tender offer game. Thus, if a fully revealing equilibrium exists, it must be that P(X) = X.

The strategy for finding such an equilibrium is to search for a pair of response functions

<sup>&</sup>lt;sup>14</sup>For each fixed (r, P),  $-\frac{\partial \Pi / \partial r}{\partial \Pi / \partial P}$  is strictly monotone in X.

q(r, P) and  $\gamma(r, P)$  such that some bid (r, X) maximizes the bidder's expected profit  $\Pi(r, P) = q(r, P) [\Phi(X) + \gamma(r, P) (X - P)]$  for every type  $X \in \mathcal{X}$ . A sufficient condition for such a pair to exist is that (a)  $\Phi'(X) \ge 0$ , i.e. private benefits are non-decreasing in security benefits, or (b)  $|\Phi(X) - \Phi(x)| \le 0.5 |X - x|$ , i.e. differences in private benefits are of a sufficiently smaller magnitude than differences in security benefits.

**Proposition 4** Given condition (a) or (b) is satisfied, the probabilistic tender offer game with exogenous private benefits has infinitely many fully revealing equilibrium outcomes. In any equilibrium, the bidder offers (r(X), X), expects to attract  $\gamma(X) \in [0.5, r(X)]$ shares, and the takeover succeeds with probability

$$q(X) = \exp\left\{-\int_{X}^{\overline{X}} \frac{\gamma(u)}{\Phi(u)} du\right\}.$$
(1)

In any equilibrium, the highest type succeeds with certainty,  $q(\overline{X}) = 1$ . Lower types pay a smaller price but face a higher failure probability. As a bidder forgoes private benefits in case of failure, this creates a trade-off between takeover probability q and per-share discount P - X: a higher type either bids for the shares at a fair price and succeeds with a high probability, or bids for them at a discount but succeeds with a lower probability. Too large a difference in takeover probability induces her to remain truthful. A sufficiently small q can hence credibly signal a low type. The rate at which the takeover probability  $q(\cdot)$  increases in the bidder's type,  $q'(X) = \gamma(X)/\Phi(X)$ , reflects the benefits and the costs of mimicking a marginally lower type. On the one hand, q'(X) increases in  $\gamma(X)$ . Larger transaction sizes make mimicking a lower type more attractive, so that decreases in takeover probability must be larger to be an effective deterrent. On the other hand, q'(X) decreases in  $\Phi(X)$ . Larger private benefits render it more costly to fail, so that smaller decreases in takeover probability are sufficient to deter mimicking.

The equilibrium outcome is not unique because there exist infinitely many pairs of response functions that satisfy this equation.<sup>15</sup> For any choice of  $\gamma(\cdot)$ , separation is achieved through the corresponding equilibrium takeover probabilities  $q(\cdot)$  determined by equation (1). Bid restrictions (or cash-equity offers) which directly affect, if at all, only  $\gamma(\cdot)$  therefore play a redundant signaling role. However, they affect the efficiency of the equilibrium outcome, as lower  $\gamma(\cdot)$  generally increase the takeover probability. Indeed,  $q'(\cdot)$  goes to 0 as  $\gamma(\cdot)$  approaches 0. That is, reducing expected transaction sizes promotes takeover activity, thereby benefiting both the bidder and the target shareholders. Thus, the most efficient, Pareto-dominant outcome is achieved when all types restrict their bid as far as possible (r = 0.5), which is the outcome selected by Hirshleifer and Titman (1990).

<sup>&</sup>lt;sup>15</sup>Also, the equilibrium outcome is only characterized by aggregate response functions but does not pin down a profile of individual shareholders' tendering strategies. That is, even for a given pair of reponse functions, there exist multiple equilibria.

**Corollary 2** In the unique Pareto-dominant fully revealing equilibrium outcome, all bidders restrict their bid to half the shares.

Restricting every type's offer to half the shares may not always be incentive-compatible. The probability decrease needed to deter high types from mimicking low types may be so large that, conversely, low types have an incentive to mimic high types. This is never the case when at least one of the conditions (a) and (b) is satisfied. When both conditions are jointly violated, fully revealing equilibria either do not exist or require that bid restrictions vary across bidder types. (We discuss this case more formally in Appendix B.) This also implies that Proposition 4 does not readily extend to the case of non-deterministic private benefits, where the bidder's type is a point in the  $[0, \bar{\Phi}] \times \mathcal{X}$  space. In this case, we can always find subsets of  $[0, \bar{\Phi}] \times \mathcal{X}$  that violate conditions (a) and (b). In fact, we show in Section 4.2.2 that the separating equilibrium is not robust when the signaling problem is two-dimensional.

# 4 Sharing Private Benefits

The probabilistic separating equilibrium suggests a mechanism to overcome the information asymmetry between a bidder and free-riding shareholders. Since P = X, the shareholders appropriate all the gains in security benefits, depriving the bidder of the possibility to forgo (part of) these gains to signal her type. But compared to the deterministic setting, the uncertainty of the tender offer outcome affects the bidder's private gains. This enables lower types to reveal themselves by accepting a higher failure probability, or equivalently, by forgoing a larger part of their expected private benefits. More generally, as the bidder cannot affect the allocation of the gains in security benefits, she manipulates her private benefits in a way that allows inference about the security benefits. (In fact, the separating equilibrium collapses when  $\Phi(\cdot) = 0$ .)

Subsequently, we show that this principle can also be applied in the deterministic setting. To this end, we examine different means by which the bidder can manipulate her private benefits. One possibility is to adopt a corporate charter or governance structure which limits the amount of private benefits the bidder can extract. Alternatively, if some takeover gains accrue to assets (other than the target) under the bidder's control, she may also grant target shareholders claims to those assets.

## 4.1 Governance as a Signal

We endogenize the private benefits by letting the bidder choose what fraction  $\phi$  of the post-takeover value  $V \in \mathcal{V}$  to divert for private consumption. Diversion does not dissipate value, so that a successful bid generates security benefits  $X(V) = (1 - \phi)V$  and private benefits  $\Phi(V) = \phi V$ . Importantly, we assume that the bidder can constrain her

ability to extract private benefits (over and above external legal constraints) through selfimposed governance provisions (Chhaochharia and Laeven, 2007). Formally, the bidder can publicly commit to any extraction rate  $\phi \in [0, \bar{\phi}]$  at the time of the bid, where  $\bar{\phi}$  is an exogenous limit set by e.g., shareholder protection laws.

The timing of the tender offer game changes accordingly. In stage 0, the bidder privately learns her type  $V \in [0, \overline{V}]$ . In stage 1, she then decides whether to make a take-it-or-leave-it, conditional, restricted cash offer. If she does not make a bid, the game moves immediately to stage 3. Otherwise, she offers to purchase a fraction r of the outstanding shares at a price rP. In addition, she chooses and announces an extraction rate  $\phi \in [0, \overline{\phi}]$ . In stage 2, the target shareholders non-cooperatively decide whether to tender their shares. In stage 3, the incumbent manager remains in control if the fraction of tendered shares  $\beta$  is less than 50 percent. Otherwise, the bidder gains control, improves the target, and extracts a fraction  $\phi$  of the value improvement as private benefits.

The tender offer is now represented by a triple  $(r, \phi, P)$ . If  $\phi$  is uniform across all bidder types, the setting becomes equivalent to the deterministic tender offer game with exogenous private benefits, in which case no separating equilibrium exists. Thus, r and P alone are not viable signals. By contrast, if  $\phi$  can be chosen to signal the bidder's type, we know from section 3.2 that r becomes a redundant signal, and that choosing r = 0.5yields the most efficient separating equilibrium outcome by minimizing the mimicking incentives of high-valued bidders. We therefore restrict our attention to bids of the form  $(0.5, \phi, P)$ .

The bidder chooses the extraction rate  $\phi \in [0, \overline{\phi}]$  and the bid price P to maximize  $\Pi = \phi V + 0.5 [(1 - \phi)V - P]$  subject to the free-rider condition  $P \ge X$ . Let us first abstract from the additional constraint imposed by the free-riding behavior, and focus purely on the non-mimicking constraints. If a fully revealing equilibrium exists, it must be implementable as a direct truth-telling mechanism. Formally, there must exist functions  $\phi(\cdot)$  and  $P(\cdot)$  such that the solution to bidder's maximization problem

$$\max_{\hat{V} \in \mathcal{V}} \left\{ \phi(\hat{V})V + 0.5[(1 - \phi(\hat{V}))V - P(\hat{V})] \right\}$$

is  $\hat{V} = V$  for every  $V \in \mathcal{V}$ , where  $\hat{V}$  is the bidder's self-reported type. The functions  $\phi(\cdot)$  and  $P(\cdot)$  must therefore satisfy the first-order condition

$$0.5\phi'(V)V = P'(V).$$
 (2)

In the Appendix, we show that equation (2) is sufficient to ensure truth-telling if  $\phi'(V) > 0$ , which in turn implies P'(V) > 0. Thus, the bidder can reveal a low type by choosing stricter governance provisions that reduce her ability to extract private benefits, and this in turn allows her bid to succeed at a lower price. Although a high-valued bidder

would like to mimic the low bid, she is deterred by the strict governance provisions as the diversion benefits increase in the total post-takeover value V.

Since equation (2) holds for many  $\phi$ -*P*-schedules, it does not pin down a unique outcome. Imposing the free-rider condition eliminates some, but not all, incentive-compatible schedules. For instance, there exists an equilibrium schedule, in which every type offers a price equal to the post-takeover share value, i.e.  $P(V) = [1 - \phi(V)]V$ . Differentiating and substituting into the first-order condition yields a differential equation for the equilibrium  $\phi$ -schedule:  $\phi'(V) = \frac{1-\phi(V)}{1.5V} > 0$ . This  $\phi$ -*P*-schedule can be supported as an equilibrium outcome by appropriately chosen out-of-equilibrium beliefs: The highest type chooses the maximum extraction rate  $\phi$  and offers the price  $\overline{X} = (1 - \phi)V$ , thereby extracting her full information profit  $\phi V$ . For other types, mimicking the highest type's equilibrium bid yields the highest payoff of any (off-equilibrium) bid with  $P \geq \overline{X}$ . By contrast, any off-equilibrium bid with  $P < \overline{X}$  fails so long as shareholders attribute it to the highest type. Compared to either alternative, the bidder prefers her own equilibrium bid, so that a Perfect Bayesian Equilibrium exists.

In equilibrium, some types make a fully revealing bid, while others do not make a bid at all. As lower types must reduce their extraction rate to provide a credible signal, there exists a cut-off type at which the incentive-compatible extraction rate falls to 0. Indeed, by the envelope theorem, the bidder's equilibrium profit increases in her type at the rate  $\partial \Pi^*/\partial V = 0.5\phi(V) + 0.5 > 0$ . Given that the highest type's profit provides an upper bound,  $\bar{\phi}\overline{V}$ , we can define the cut-off type  $V^c(\bar{\phi})$  by

$$\int_{V^c(\bar{\phi})}^{\overline{V}} \left\{ 0.5\phi(u) + 0.5 \right] \right\} du = \bar{\phi}\overline{V}.$$

**Proposition 5** In the tender offer game where bidders can commit to an extraction rate  $\phi \in [0, \bar{\phi}]$ , a fully revealing equilibrium exists. All types above the cut-off type  $V^c(\bar{\phi}) \in [0, \bar{V})$  make a bid for half the shares offering P(V) = X(V), and higher types extract a larger fraction of V as private benefits.

The equilibrium exhibits an unexpected property that epitomizes the signaling role of private benefits. As both  $\phi(\cdot)$  and  $P(\cdot)$  are increasing, private benefit extraction and bid premia are positively correlated in equilibrium: a bidder who extracts a larger fraction of the proceeds V as private benefits is associated with a higher post-takeover share value, and hence pays a higher price. Or putting it differently, acquirers with worse corporate governance create more value for the target (shareholders). Interestingly, our result suggests that empirical findings that acquirers with weaker corporate governance pay higher bid premia need not necessarily imply that they overpay (more) as a result of agency problems, and the takeover is therefore less efficient. On the contrary, in our model, such takeovers are the *most* efficient. As in section 3.1, the cut-off type is decreasing in the overall scope for private bidder gains, which is in this case determined by the quality of shareholder protection laws. Better legal protection reduces the extent to which the bidder can privately manipulate its governance quality to signal her type. More generally, the less value the bidder can (re)allocate through the takeover design, the smaller is her signaling power. In the limit, as  $\bar{\phi}$  approaches 0,  $V^c(\bar{\phi})$  converges to  $\bar{V}$ , and takeover activity (except by the highest type) breaks down.

Firm-level governance provisions are, of course, not the only means of manipulating private bidder gains. In fact, the same logic can be applied to any source of exclusionary, yet manipulable, benefits. For instance, the above analysis remains almost entirely unchanged, if one simply replaces the extraction rate  $\phi$  with a toehold  $\alpha \in [0, \bar{\alpha}]$  that the bidder can acquire at the current share value (i.e., without a price impact). The upper bound  $\bar{\alpha}$  in this case may represent a mandatory disclosure, or mandatory bid, rule that prevents the bidder from acquiring an even larger pre-bid stake in the target.<sup>16</sup>

Another viable signal is to raise debt against the targets assets in financing the bid, which can serve as a substitute for private benefit extraction (Panunzi and Müller, 2002). Using less debt-financing, and thereby extracting less of the target value, is also a credible way to signal a low type. A high type does not mimic a low type because the gains from purchasing the shares at a discount are offset by the decrease in leverage, whereas the low type does not mimic the high type because the increase in leverage is offset by the premium she must pay for the shares. For example, consider two types L and H, with  $X_L = 80$  and  $X_H = 100$ , and the offer schedule  $r_L = r_H = 1$ ,  $D_L = P_L = 40$ ,  $D_H = 60$ and  $P_H = 40$ . H's non-mimicking constraint is  $60 \ge 40 + (60 - 40)$ . While H can purchase target shares at a discount of (60 - 40) by mimicking L, she must also lower her leverage from 60 to 40. Similarly, L's non-mimicking constraint is  $40 \ge 60 - (40 - 20)$ . While Lcan raise her leverage from 40 to 60 by mimicking H, she must also pay a bid premium of (40 - 20). In either case, the gains and losses cancel out.

## 4.2 Bidder Assets and Mergers

In section 3.1, we show that cash-equity offers cannot reveal a bidder's type when the private benefits are exogenously given (Corollary 1). While we have stressed the role of the free-rider condition, the result is also due to the absence of bidder assets (other than the target). In an extended setting with bidder assets A > 0, the bidder can use claims to such assets to pay target shareholders, and the willingness to do so may reveal her type. As we show below, the viability of such signals crucially depends on the correlation between the bidder assets A and the target's value improvement X. We therefore examine

<sup>&</sup>lt;sup>16</sup>The signaling potential of toeholds has been analyzed within a probabilistic tender offer game by Chowdhry and Jegadeesh (1990). Our analysis highlights that toeholds are a particular implementation of a general principle, and that the implementation does not require a probabilistic setting.

in turn the cases of perfect and imperfect correlation.

#### 4.2.1 Perfect Correlation

Cash aside, the bidder now owns a separate firm whose security benefits are  $A(X) \equiv Z + \lambda X$ , where  $Z \geq 0$  and  $\lambda \geq 0$  are commonly known and the same for all types. We consider the case of a takeover in which the bidder combines her own assets and the acquired target stake in a holding company H. Target shareholders are offered a cash price  $C(\beta)$  and  $t(\beta)$  shares in the holding company, where  $\beta$  is the fraction of shares tendered. To merge the firms, the bidder makes a bid for all target shares, i.e. r = 1. We further assume that target shareholders are cash-constrained, and that the bidder is unwilling to relinquish majority control of the holding company. This imposes restrictions on the set of admissible offers, modeled as a *cash* constraint  $C(\beta) \geq 0$  and a *control* constraint  $t(\beta) \in [0, 0.5]$ . The timing of the tender offer game remains the same as in section 2.

If the bid succeeds ( $\beta \ge 0.5$ ), the holding company is worth  $H(\beta, X) = A(X) + \beta X$ . Under full information, free-riding target shareholders therefore do not tender their shares unless  $C(\beta) + t(\beta)H(\beta, X) \ge X$ . To ensure a successful merger ( $\beta = 1$ ), the bidder must choose  $t(\beta)$  and  $C(\beta)$  such that

$$t(\beta) \ge \frac{X - C(\beta)}{Z + \beta X + \lambda X} \tag{3}$$

for all  $\beta \in [0.5, 1]$ . In this case, all shareholders tender their shares whenever they believe that more than half the shares are tendered, and the bidder must ultimately pay C(1) and t(1). To simplify the exposition, we therefore suppress the contingent nature of the offer, and express the bidder's offer as a pair (C, t) which must satisfy the free-rider condition for  $\beta = 1$ , i.e.  $t \ge (X - C)/(Z + X + \lambda X)$ .<sup>17</sup> Note that condition (3) violates neither the cash constraint nor the control constraint if  $C(\beta)$  is chosen sufficiently high.

For a given cash price C and equity component t, the bidder's payoff from a successful merger is  $\Pi = (1-t)H(X) - C$ . As before, we formulate the bidder's problem as a direct mechanism, in which she solves

$$\max_{\hat{X}\in\mathcal{X}}\left\{ [1-t(\hat{X})]H\left(X\right)-C(\hat{X})\right\} .$$

We show in the Appendix that a fully revealing equilibrium requires  $t'(\cdot) < 0$  and -t'(X)H(X) - C'(X) = 0, which in turn imply that  $C'(\cdot) > 0$ . Together, these conditions ensure quasi-concavity of the objective function. That is, they are jointly necessary

<sup>&</sup>lt;sup>17</sup>Even without a contingent offer, there exists a self-fulfilling equilibrium in which the merger succeeds for (C, t) as long as it satisfies the free-rider condition for  $\beta = 1$ : If each shareholder believes that all other shareholders tender, she also tenders. Hence, once can alternatively focus on non-contingent offers, and select merger success as the equilibrium outcome whenever it is consistent with the free-rider condition.

and sufficient for incentive-compatibility.

In equilibrium, lower bidder types pay lower cash amounts but give up more of postmerger equity. As before, we use the envelope theorem to determine how takeover profits vary in equilibrium; and show that  $\partial \Pi^* / \partial V = [1 - t(X)](1 + \lambda)$  is positive for some types below  $\overline{X}$ , who in equilibrium pays a zero premium. Thus, unless the bidder enjoys some exclusionary gains, it is not worthwhile to signal a low type. The mere fact that the bidder assets are informative about the target value improvement is hence insufficient to obtain a fully revealing equilibrium. The bidder must also enjoy private takeover gains.

We therefore assume synergy benefits S > 0 that accrue to the bidder assets if the merger succeeds. Whether S is correlated with X, or is subsumed in A(X), is not crucial for the results. What is important is that S only materializes if the merger succeeds, and that shareholders can only participate in this gain if they tender and get shares in the post-takeover firm. For simplicity, let S thus be a constant subsumed in Z.

Given the profit slope, and the highest type's equilibrium profit S, similar calculations as before show that there may exist a type  $X_{\pi}^{c}(S) \in [0, \overline{X})$  below which incentivecompatible profits become negative. However, in this case, this need not be the binding constraint. As lower types issue more equity, they may also run into either the control constraint  $t(\cdot) \leq 0.5$  or the cash constraint  $C(\cdot) \geq 0$ . The latter may occur because the bidder can in principle become a net *issuer*, rather than a net purchaser, of claims. The cash constraint is particularly severe for bidders that have many unrelated assets Z.

Which of the three constraints (participation, cash or control) is binding – and hence which among  $X_{\pi}^{c}$ ,  $X_{t}^{c}$  and  $X_{C}^{c}$  is the true cut-off type – depends on the particular schedule chosen to satisfy  $t'(\cdot) < 0$  and -t'(X)H(X) - C'(X) = 0.

**Proposition 6** In the tender offer game with perfectly correlated bidder assets A and target value improvement X, a fully revealing equilibrium exists. All types above the cutoff type  $\max\{X_{\pi}^{c}, X_{C}^{c}, X_{t}^{c}\} \in [0, \overline{V})$  make a merger bid, and lower types pay more in equity of the merged company.

The key is that paying in shares of the merged firm allows the bidder to concede a part of her private synergy gains S, in a manner which is informative about X. Thus, a higher equity component in this case is analogous to a lower  $\phi$  in section 4.1. It is important that S are private benefits or synergy gains that accrue to the bidder company, and not to the target. Crucial to the bidder's signaling ability is therefore how the synergy gains are divided between the two firms. (For example, a merger between a soft drink producer and a fast food chain may have separable effects on the sales in each firm.)

The equilibrium schedule in Proposition 6 is similar to those found in the literature on the means of payment in bilaterally negotiated mergers. Though contrary to bilateral merger models, tender offer games do not require two-sided asymmetric information to generate a role for cash-equity offers involving bidder assets. It is enough that the bidder has private information about the post-takeover value improvement in the target.

#### 4.2.2 Imperfect Correlation

The setting in the previous section is special, which casts doubt on the generality of Proposition 6. First, the assumed perfect correlation between A and X is not only restrictive but may prove crucial. To give the simplest example, if the bidder assets were known to have a constant value,  $A(\cdot) = A$ , they would be equivalent to cash and therefore have no signaling value. Second, the assumption that the bidder must merge the target firm with her own assets reduces the set of offers that the bidder can make. In essence, it does not allow the bidder to separately issue equity in the target firm and in the bidder assets after the takeover.

In this section, we therefore analyze a more general case. The bidder's type is now twodimensional, (X, A), and continuously distributed on the space  $[\underline{X}, \overline{X}] \times [\underline{A}, \overline{A}]$  according to some joint density function. For simplicity, we assume that A consists entirely of synergy gains. In the previous notation, this means that A = S. The bidder is privately informed about both dimensions of her type. That is, she knows the post-takeover target value and the post-takeover value of her own assets.

In addition, the bidder can pay target shareholders a fraction  $t \in [0, 0.5]$  of the bidder assets (only) and restrict her offer to  $r \in [0.5, 1]$  target shares. Thus, a tender offer is a triple (r, t, P). This setting subsumes the merger case where r = t.

To examine the existence of a fully revealing equilibrium, consider the type  $(\overline{X}, \underline{A})$  and an arbitrary type  $(X, A) \neq (\overline{X}, \underline{A})$ . In a fully revealing equilibrium, type  $(\overline{X}, \underline{A})$  cannot be held to a profit lower than  $\underline{A}$  because she can always succeed with the bid  $(1, 0, \overline{X})$ . At the same time, she cannot earn more than  $\underline{A}$  because of the free-rider condition, which ensure that the target shareholders get at least  $\overline{X}$ . In order for type  $(\overline{X}, \underline{A})$  not to mimic type (X, A), the latter must make an offer (r, t, P) which satisfies  $\underline{A} \ge r\overline{X} + (1-t)\underline{A} - C$ , or equivalently

$$C \ge \underline{C} \equiv r\overline{X} - t\underline{\Phi}.\tag{4}$$

In addition to this incentive-compatibility constraint, the (conjectured) truthful offer by (X, A) must also yield a higher profit than the "off-equilibrium" offer  $(0.5, 0, 0.5\overline{X})$ which succeeds irrespective of target shareholder beliefs. That is, her offer (r, t, P) must satisfy  $rX + (1 - t)A - C \ge 0.5(X - \overline{X}) + A$ , or equivalently

$$C \le \overline{C} \equiv (r - \underline{r}) X + \underline{r} \overline{X} - t\Phi.$$
(5)

The key question is whether there exist an offer (r, t, P) that simultaneously satisfies both constraints (4) and (5). For this to be possible, it must be that  $\overline{C} \geq \underline{C}$ . After substituting the full expressions, straightforward manipulations yield  $(r - 0.5)(X - \overline{X}) \ge t(A - \underline{A})$  which is always violated. Thus, all types  $(X, A) \neq (\overline{X}, \underline{A})$  prefer the offer  $(0.5, 0, 0.5\overline{X})$  over any offer that separates them from type  $(\overline{X}, \underline{A})$ . Thus, there exists no fully revealing schedule.

**Proposition 7** In the tender offer game in which the value of the bidder's assets A are not informative about the target value improvement X and are private information, no fully revealing equilibrium exists.

The fundamental reason why signaling breaks down in the two-dimensionsal case is that the private information about A undermines the "trustworthiness" of t (i.e., the payment in bidder shares) as a signal. Intuitively, when A is not a deterministic function of X, the target shareholders cannot assess how costly it is for the bidder to concede bidder shares. The uncertainty about A "jams" the signal.

The assumption that the bidder knows more about her own assets than dispersed target shareholders seems reasonable in most real-world settings. In this light, Proposition 7 reinforces the conclusion from Corollary 1 that cash-equity offers are a poor signaling device in tender offers. Finally, it should be noted that the above insight is not confined to the setting with bidder assets. Any signaling equilibrium which relies on the bidder's forgoing of private benefits, including the probabilistic equilibrium of Section 3.2, is likely to collapse when the bidder has private information about her private benefits.

## 5 Separating Votes and Cash Flows

The signaling equilibria analyzed so far exhibit inefficiency at the "bottom". The probabilistic equilibria cause lower types to fail with higher probability, whereas the deterministic equilibria make a bid too costly for types below a certain threshold. A common feature of all these equilibria is that more takeovers materialize, the fewer shares the bidder needs to acquire to gain control. The reason is that smaller transaction sizes mitigate the adverse selection problem, i.e. the bidder's incentives to mimic a lower type. Bid restrictions – though redundant as a signal – hence promote an efficient control allocation.

Efficiency would be further improved if the bidder could gain majority control without having to acquire a majority stake. In contrast to security issuance models (e.g., Myers and Majluf, 1984; DeMarzo and Duffie, 1999), the gains from trade in tender offer models are typically contingent on the transfer of votes, and not of the cashflow rights *per se*.<sup>18</sup> This provides an alternative view of the adverse selection problem in tender offer games:

<sup>&</sup>lt;sup>18</sup>This is not true if the bidder's incentives to improve the target value vary with her post-takeover stake. Separating cashflow and voting rights can in this case be beneficial even in the absence of asymmetric information (Burkart et al., 1998).

value-improving control transfers are impaired by the fact that control must be transferred in conjunction with misvalued cashflow rights. The logical solution is therefore to unbundle cashflow and votes.<sup>19</sup>

## 5.1 Dual-Class Offers

Perhaps the most straightforward way to unbundle control and ownership is to make a dual-class security-exchange offer. In a dual-class offer, the bidder offers to exchange each of the target's voting shares against a non-voting share. Shareholders accept the bid as long as it preserves their fraction of the cashflow rights. If the offer were to exchange shares at less than a one-to-one ratio, each shareholder would reject it. By construction, the bidder pays exactly the post-takeover security benefits to gain control. This replicates the full information outcome without revealing the bidder's type.

Although the dual-class offer resolves the asymmetric information problem, it is problematic because it leaves all cash flow rights with the shareholders. That is, the bidder has no equity interest in the firm after the takeover. On the one hand, this makes the dual-class offer equivalent to a simple replacement of management, which begs the question why a takeover is needed in the first place. On the other hand, it makes such offers prone to abuse by value-decreasing bidders (or "fly-by-night" operators), since it does not require the bidder to put up any cash (Bebchuk and Hart, 2001). Cash payments put at least some (lower) bounds on the bidder's quality.

## 5.2 Derivatives

Dual-class offers or other extreme solutions, which leave the bidder with no equity interest in the firm ex post, are unnecessary. The bidder is merely unwilling to pay for cashflows which she knows do not to exist. The ideal solution is therefore to let the bidder acquire the target in exchange for cash and a set of securities which leave the "non-existing" cashflow to target shareholders. Such a trade can be implemented with call options. It merely requires that every type  $X \in \mathcal{X}$  purchases a target share in exchange for cash X and a call option with a strike price of X. To see that this is incentive-compatible, consider two arbitrary types X and x with X > x. If the high type mimics the lower bid, she ex ante pays a cash price x for shares that are worth X. However, ex post she cannot capitalize on this gain, as the target shareholders will exercise their options once the actual value improvement becomes known to the market. Conversely, the low type does not mimic the high type because she would pay X for shares that are worth x. Thus, the offer schedule is incentive-compatible. Moreover, every bidder type succeeds, irrespective of how many shares she acquires or whether she enjoys any private benefits.

<sup>&</sup>lt;sup>19</sup>Based on the same conclusion, At et al. (2008) show that it may be socially and privately optimal for the target firm (shareholders) to adopt a dual-class share structure.

**Proposition 8** Suppose that the bidder can purchase a target share in exchange for a combination of cash and a call option. There exists a fully revealing equilibrium in which the cash price is P(X) = X and the strike price of the option is S(X) = X, and it coincides with the full information outcome.

Financial engineering allows the bidder (i) to trade economic ownership void of voting rights and (ii) to issue contingent claims. The first part allows the bidder to acquire the target shares, strip them of their votes, and reissue the cashflow rights. The second part allows the bidder to issue claims that punish her for "lying" about her security benefits. In particular, call options that are executed when the post-takeover security benefits are higher than professed penalize the pretense of low security benefits – ex post when the true value is observed. This makes the offer schedule in Proposition 8 equivalent to the most straightforward solution to the adverse selection problem: a bid price which is contingent on the post-takeover share value.

More generally speaking, this is a security design solution. A comparison to models of external financing under adverse selection (e.g., Myers and Majluf, 1984; Duffie and DeMarzo, 1999) makes this clear. In those models, the security issuer may want issue debt to signal a high value. That is, she sells off low cashflow realizations and retains (part of the) high cashflow realizations.<sup>20</sup> By contrast, in tender offer games, the bidder wants to signal a low value. Hence, she does the opposite: she buys the target, retains low cash flow realizations, and reissues high cash flow realizations to the target shareholders.

The above solution transfers, cash apart, only cash flow claims but no actual future cash flow to target shareholders. This is an artefact of the assumption that the posttakeover share value X is deterministic (and perfectly known by the bidder). When X is a random variable, a bidder can still signal her type if there exists a set of states for which her expected cashflow is higher than for any other type. For example, consider three types  $i \in \{H, M, L\}$  with  $X \sim N(\mu_i, \sigma)$  and  $\mu_H > \mu_M > \mu_L$ . Figure 1 depicts their probability density functions.

<sup>&</sup>lt;sup>20</sup>Strictly speaking, the optimality of debt requires certain regularity conditions on the distribution of the cashflow realizations.



Figure 1: The 3-type example

A possible fully revealing equilibrium is that L retains only cash flows in  $[\infty, S_1]$ , Mretains only cash flows in  $[S_1, S_2]$ , and H retains only cash flows in  $[S_2, \infty]$ . This can be implemented through combinations of cash and (put and call) options with strike prices of  $S_1$  and  $S_2$ . For example, M then acquires all cash flow realizations  $X \in [S_1, S_2]$ against a cash price of  $P(M) = E_M(X | S_1 \leq X \leq S_2)$ . Since  $E_M(X | S_1 \leq X \leq S_2) >$  $E_H(X | S_1 \leq X \leq S_2) > E_L(X | S_1 \leq X \leq S_2)$ , H or L would make a loss when mimicking M's bid, whereas they break even under a truthful bid. The same reasoning explains why the other two equilibrium bids credibly reveal the bidder's type as L or H.

Although theoretically appealing, unbundling control and ownership can in practice be difficult to implement or may create other problems (Hu and Black, 2006, 2008). As already mentioned, our analysis takes the potential value improvement as given, and hence abstracts from the bidder's post-takeover incentives to improve the target value. These incentives would certainly depend on the nature and the magnitude of the economic interest that she retains in the firm. This is of particular concern as the bidder's payoff under the proposed schedule, in the case where X is stochastic, is non-monotonic in X. For example, when X is above  $S_1$ , M has strong incentives to decrease the share value.

## 6 Conclusion

This paper analyzes tender offers in which a single bidder is better informed about the post-takeover share value than dispersed target shareholders. Two key features of the tender offer process render this situation very different from standard bilateral trade models. First, free-riding shareholders have full bargaining power over the value improvement in the target shares, even though the better informed bidder makes a take-it-or-leave-it offer. Second, the parties in a tender offer bargain both over control (voting rights) and over ownership (cash flow rights) in the target firm. That is, unlike other signaling models

in finance, a share (trade) represents a (trade of a) bundle of two goods with potentially distinct values.

We demonstrate that these differences lead to constraints as well as solutions that are absent in bilateral trade models. Because the bidder is forced to concede all gains in share value to the shareholders, she cannot signal her type by *voluntarily* giving up such gains. Neither restricted bids nor cash-equity offers are therefore viable signals in tender offers. Instead, the bidder must enjoy private benefits that are not only excluded from bargaining but can also be forgone in a manner which allows inference about the post-takeover share value. Firm-level governance provisions which limit private benefit extraction, debt financing or toeholds can serve this purpose. The underlying principle in all cases is the same: the bidder must forgo (more) private benefits to signal a low(er) type. Unfortunately, some low-value bidders may find it too costly to signal their type even if the takeover would be efficient.

Such inefficiencies can be overcome if the bidder can include derivatives in the tender offer terms. Derivatives allow the bidder to separate cash flow rights from voting rights. This separation prevents that the information problems in the trade of cash flow rights spill over into, and thereby impair, the trade of voting rights. As a result, control can be transferred efficiently irrespective of any disagreement between the bidder and the target shareholders about the value of the post-takeover cash flow rights.

Our analysis has implications for the design of takeover bids. For instance, it suggest that derivatives as a means of payment should play a more prominent role in tender offers than the combination of cash and equity. Furthermore, acquiring firms may signal their quality through self-imposed governance provisions or the amount of takeover leverage. The main theoretical contribution of this paper is to study how the interaction of asymmetric information and collective action problems, in a specific market setting, may bear on the optimal design of a trade contract. We believe that there are many situations other than tender offers in which such interactions are potentially important.

# Proofs

## **Proof of Corollary 1**

Corollary 1 follows from the equivalence of mixed offers and restricted cash-only offers which the subsequent lemma establishes. Consider a bid for r target shares that offers a cash price C and t shares in the post-takeover firm.

**Lemma 1** Under full information, the restricted mixed offer (r, C, t) and the restricted cash-only offer  $(r^{co}, C^{co})$  with  $C^{co} = C$  and  $r^{co} = r - t$  are payoff-equivalent.

**Proof.** To succeed, the mixed offer must satisfy the free-rider condition  $C + tX \ge rX$ , or equivalently

$$C/r + (t/r) X \ge X. \tag{6}$$

Given the condition is satisfied, all shareholders tender, and the bidder's payoff is

$$\Phi(X) + r \left[ X - (C/r + (t/r)X) \right].$$
(7)

Rearranging the free-rider condition (6) to

$$C \ge (r-t) X$$

and the bidder's payoff (7) to

$$\Phi(X) + (r-t)X - C$$

shows that the restricted cash-only offer  $(r^{co}, C^{co})$  with  $C^{co} = C$  and  $r^{co} = r - t$  is payoff-equivalent for any X.

Hence, if a fully revealing equilibrium in mixed offers were to exist, a fully revealing equilibrium in cash-only offers would also exist. As Proposition 1 rules out the latter, a mix of cash and equity is not a viable signal. ■

## **Proof of Proposition 2 and Proposition 3**

We begin by characterizing the general properties of an incentive-compatible r-P-schedule (Lemma 2). Then we implement the schedule for the game with bidder bargaining power and derive the cut-off type  $X^{c}(\omega)$ .

**Lemma 2** In a fully revealing equilibrium,  $r(\cdot)$ ,  $P(\cdot)$  and  $\Pi(\cdot)$  must increase with the bidder's type.

**Proof.** Without loss of generality, choose an arbitrary pair of types, X and x, and let X > x. A fully revealing schedule  $\{(r(\cdot), P(\cdot))\}$  must satisfy the non-mimicking constraints

$$r(X)[X - P(X)] \ge r(x)[X - P(x)] \quad \text{for } (x, X) \in \mathcal{X}^2.$$
(8)

We first show by contradiction that (8) requires r(X) > r(x). The non-mimicking constraints for type X and x are respectively

$$C(X) - r(X)X \le C(x) - r(x)X$$
 and  $C(x) - r(x)x \le C(X) - r(X)x$  (9)

where  $C(\cdot) \equiv r(\cdot) P(\cdot)$ . For r(X) = r(x), the inequalities hold jointly only if C(X) = C(x), and hence P(X) = P(x), in which case the two offers would be identical. For r(X) < r(x), rewrite (9) as

$$C(x) \ge C(X) + [r(x) - r(X)] X$$
 and  $C(x) \le C(X) + [r(x) - r(X)] x$ .

Since C(X) + [r(x) - r(X)] X > C(X) + [r(x) - r(X)] x, the constraints cannot hold jointly. Thus, the non-mimicking constraints are violated unless  $r(\cdot)$  is increasing. Given r(X) > r(x), condition (8) implies that the bid price and the bidder's profit must also be increasing in her type. To this end, we rewrite (9) as

$$r(X)[X - P(X)] \ge r(x)[X - P(x)]$$
 and  $\frac{r(X)}{r(x)}[x - P(X)] \le x - P(x)$ .

Given that r(X)/r(x) > 1, the second inequality implies P(X) > P(x). Furthermore, as r(x)[X - P(x)] > r(x)[x - P(x)], the first inequality implies  $r(X)[X - P(X)] \ge r(x)[x - P(x)]$ . Thus, higher types must pay higher prices and make higher profits.

Lemma 2 states necessary conditions for incentive-compatibility. To derive sufficient conditions, we now impose that  $r(\cdot)$  and  $C(\cdot)$  are continuously differentiable functions, and cast the bidder's optimization as a direct mechanism:

$$\max_{\hat{X}\in\mathcal{X}}\left\{r(\hat{X})X-C(\hat{X})\right\}$$

In equilibrium, the first-order condition must hold at  $\hat{X} = X$ , i.e.

$$r'(X)X = C'(X).$$
 (10)

Condition (10) is sufficient to ensure incentive-compatibility if the above maximization problem is quasi-concave (and shareholders' out-of-equilibrium beliefs are suitably chosen). Substituting  $r'(\hat{X})\hat{X} = C'(\hat{X})$  into the derivative of the objective function gives

$$\frac{\partial}{\partial \hat{X}} \left[ r(\hat{X})X - C(\hat{X}) \right] \Big|_{C'(\hat{X}) = r'(\hat{X})\hat{X}} = r'(\hat{X})X - r'(\hat{X})\hat{X} = r'(\hat{X})(X - \hat{X}).$$

From Lemma 2  $(r'(\cdot) > 0)$  it follows that the derivative switches sign at most once (from positive to negative), and the objective function is strictly quasi-concave.

Condition (10) puts a constraint on how equilibrium profits  $\Pi^*(X) = r(X)X - C(X)$ can vary across different types. By the envelope theorem,

$$\frac{\partial \Pi^*(X)}{\partial X} = \underbrace{r'(X)X - C'(X)}_{=0} + r(X) = r(X).$$
(11)

That is, the marginal change in profits is given by the bid restriction r(X).

In a fully revealing equilibrium, no bidder type can make higher profits than under full information. In the game with bidder bargaining power, this means that bidder profits are bounded from above by  $\omega X$ . This additional constraint on the incentive-compatible schedule is, for instance, satisfied if one chooses  $r(\cdot)$  such that equilibrium profits decrease at a larger rate than under full information (which is  $\omega$ ). By choosing  $C(\cdot)$  appropriately, we can ensure that (10) continues to hold while satisfying  $r(X) > \omega$  for all X.

Given bidders have bargaining power  $\omega$ , shareholders always tender at the price  $P = (1 - \omega)\overline{X}$ . As type  $\overline{X}$  buys shares below their true value, she buys all shares and makes a profit  $\Pi^*(\overline{X}) = \omega \overline{X}$ . Since profits decrease at the rate r(X) (condition (11)), the threshold type  $X^c$ , making zero profits, is defined by

$$\int_{X^c}^{\overline{X}} r(u) du = \omega \overline{X}.$$

Clearly,  $\partial X^c / \partial \omega < 0$ , and  $\lim_{\omega \to 0} X^c = \overline{X}$ .

## **Proof of Proposition 4**

We solve the problem by first focusing on the subproblem of maximizing the bidder's profit only with respect to P. That is, initially, we assume r(X) as given for all  $X \in \mathcal{X}$ , such that it is merely a parameter that effects the values of  $q(\cdot)$  and  $\gamma(\cdot)$  (and that, for notational convenience, we thus suppress). The maximization problem is then

$$\max_{P \in \mathbb{R}} \Pi(P; X) \equiv q(P) \left[ \Phi(X) + \gamma(P)(X - P) \right].$$

A fully revealing equilibrium requires that the solution to the first-order condition,

$$q'(P) \left[ \Phi(X) + \gamma(P)(X - P) \right] + q(P) \left[ \gamma'(P)(X - P) - \gamma(P) \right] = 0,$$

is given by the truthful bid P = X for all  $X \in \mathcal{X}$ . Substituting and rearranging shows that this condition is satisfied if

$$\frac{q'(X)}{q(X)} = \frac{\gamma(X)}{\Phi(X)}.$$

This first-order differential equation for  $q(\cdot)$  has the solution

$$q(P) = A \exp\left\{\int_0^P \frac{\gamma(u)}{\Phi(u)} du\right\}$$

where A is an integration constant. From the free-rider condition it follows that for all  $P < \underline{X}$ , q(P) = 0, and for all  $P \ge \overline{X}$ , q(P) = 1. Thus, A is uniquely determined by the boundary condition,  $q(\overline{X}) = 1$ , so that

$$q(P) = \begin{cases} 1 & \text{for } P > \overline{X} \\ \exp\left\{-\int_{P}^{\overline{X}} \frac{\gamma(u)}{\Phi(u)} du\right\} & \text{for } P \in \mathcal{X} \\ 0 & \text{for } P < 0 \end{cases}$$
(12)

For  $P \in [0, \overline{X})$ , this probability schedule strictly assumes interior values, which is consistent with the free-rider condition precisely when the schedule indeed elicits truthful offers such that shareholders are indifferent. (Note also that the schedule is strictly increasing.)

So far, we have only shown that P = X satisfies the first-order condition. A sufficient condition for truthful offers to be *globally* preferred to any other equilibrium offer is that  $\Pi(P; X)$  is quasi-concave in P. To examine under which conditions quasi-concavity is satisfied, we rewrite the first-order condition:

$$q'(P) = q(P)\frac{\gamma(P)}{\Phi(P)}.$$
(13)

Substituting (13) into the derivative of  $\Pi(P; X)$  with respect to P and rearranging shows that  $\partial \Pi / \partial P > 0$  if and only if

$$\left[\Phi(X) - \Phi(P)\right] + \underbrace{\left[\gamma(P) + \frac{\Phi(P)\gamma'(P)}{\gamma(P)}\right]}_{A(P)}(X - P) > 0.$$

$$(14)$$

Suppose that condition (a) is satisfied, i.e.  $\Phi'(X) \ge 0$ . For any function  $\gamma(\cdot)$  with  $\gamma'(\cdot) \ge 0$ , which ensures that  $A(\cdot) > 0$ , the left-hand side in (14) is negative for P < X, equal to 0 for P = X, and positive for P > X. That is,  $\Pi(P; X)$  is quasi-concave.

Similarly, suppose that condition (b) is satisfied, i.e.  $|\Phi(X) - \Phi(P)| \le 0.5 |X - P|$ . In this case, the sign of the left-hand side in (14) is solely determined by A(P)(X - P) as long as  $A(P) \ge 0.5$ . Since  $\gamma(P) \ge 0.5$ , The latter inequality is satisfied for any function  $\gamma(\cdot)$  with  $\gamma'(\cdot) \geq 0$ . Thus, even if  $\Phi(\cdot)$  were decreasing, the left-hand side in (14) is negative for P < X, equal to 0 for P = X, and positive for P > X. That is,  $\Pi(P; X)$  is quasi-concave.

It should be noted that the conditionality of the bid implies that  $\gamma(\cdot) \ge 0.5$ , whereas the bid restriction implies that  $\gamma(\cdot) \ge 0.5$ . Thus, for a given restriction  $r(\cdot) \ge 0.5$ , both the expected and the actual fraction of shares tendered,  $\gamma(\cdot)$  and  $\beta(\cdot)$  respectively, can lie anywhere in the interval [0.5, r]. Though it may seem intuitive, it need not be the case that a reduction in r(X) goes together with a reduction in  $\gamma(X)$ . Though by setting r = 0.5, the bidder can pin down, indeed minimize, the fraction of shares she acquires when the bid succeeds.

In sum, if condition (a) or (b) are satisfied, there exist infinitely many schedules – corresponding to infinitely many alternatives for  $\gamma(\cdot)$  – that render the bidder's maximization problem quasi-concave, and hence a truthful offer incentive-compatible with respect to all other equilibrium offers. (Note that not all of these schedules *need* to satisfy  $\gamma'(\cdot) \geq 0$ ).

Finally, we determine the beliefs that prevent deviations from the proposed equilibrium schedule. Any out-of-equilibrium bid with  $P^o \in [0, \overline{X})$  can be made to fail, and is hence beliefs. Any out-of-equilibrium belief  $E(X | r^o, P^o) = \overline{X}$ . By contrast, no (out-ofequilibrium) offer with  $P \geq \overline{X}$  fails irrespective of shareholder beliefs. Because of the same reasons as in the proof of Proposition 9, expected equilibrium profits  $q(X)\Phi(X)$ must therefore be higher than the offer  $(0.5, \overline{X})$ . This imposes an additional constraint on the equilibrium schedule, namely  $q(X)\Phi(X) \geq \Phi(X) - 0.5(\overline{X} - X)$ , or equivalently

$$q(X) \ge 1 - 0.5 \frac{(\overline{X} - X)}{\Phi(X)} \quad \text{for all } X \in \mathcal{X}.$$
(15)

At least for any schedule such that  $r(\overline{X}) = 0.5$ , and in particular  $r(\cdot) = 0.5$ , the constraint is satisfied, as  $(0.5, \overline{X})$  is part of the equilibrium schedule (12). In fact, if the constraint is not binding for  $\underline{X}$  under  $r(\cdot) = 0.5$ , there exist infinitely many perturbations of  $r(\cdot) = 0.5$ (e.g., increasing  $r(\underline{X})$ ) that are also equilibrium schedules. In other words, there are infinitely many equilibrium outcomes unless

$$\exp\left\{-\int_{\underline{X}}^{\overline{X}} \frac{0.5}{\Phi(u)} du\right\} = 1 - 0.5 \frac{(\overline{X} - \underline{X})}{\Phi(\underline{X})}$$

or equivalently,

$$\Phi(\underline{X}) = \frac{0.5(X - \underline{X})}{1 - \exp\left\{-\int_{\underline{X}}^{\overline{X}} \frac{0.5}{\Phi(u)} du\right\}}.$$

Whether this is holds is solely determined by the form of  $\Phi(\cdot)$ . Thus, unless  $\Phi(\cdot)$  happens to have a form that satisfies this particular equation, there exist infinitely many schedules apart from  $r(\cdot) = 0.5$  that are incentive-compatible with respect to all, equilibrium and out-of-equilibrium, offers.

## **Proof of Corollary 2**

First, it follows from the proof of Proposition 4 that a constant  $\gamma(\cdot)$  makes the bidder's maximization problem quasi-concave if condition (a) or (b) are satisfied. This provided, we now show that a fully revealing equilibrium Pareto-dominates another, if the expected fraction of shares tendered in the former is weakly lower for every, and strictly lower for some, bidder type. This implies that the fully revealing equilibrium with  $\gamma(X) = 0.5$  for all  $X \in \mathcal{X}$  Pareto-dominates all others.

Consider two fully revealing equilibria,  $\mathcal{E}_1$  and  $\mathcal{E}_2$ , with  $\gamma_1(\cdot)$  and  $\gamma_2(\cdot)$ , where  $\gamma_1(X) \leq \gamma_2(X)$  for all  $X \in \mathcal{X}$  and  $\gamma_1(X) < \gamma_2(X)$  for some  $X \in \mathcal{X}$ . Since the equilibria are fully revealing, a bidder's payoff conditional on success is given by her private benefits  $\Phi(X)$  in either case. Similarly, shareholder gains conditional on success are given by the increase in the security benefits in either case. Therefore, differences in expected payoffs can only be due to differences in takeover probability. Importantly, both the bidder and the shareholders gain from an increase in takeover probability. Finally, note that

$$q_1(X) = \exp\left\{-\int_X^{\overline{X}} \frac{\gamma_1(u)}{\Phi(u)} du\right\} > \exp\left\{-\int_X^{\overline{X}} \frac{\gamma_2(u)}{\Phi(u)} du\right\} = q_2(X)$$

for all  $X \in \mathcal{X}$ , as  $\gamma_1(u) \leq \gamma_2(u)$  for all  $u \in [X, \overline{X}]$  and  $\gamma_1(u) < \gamma_2(u)$  for some  $u \in [X, \overline{X}]$ . Thus, the takeover probability of any bidder type, and hence the expected payoff of both the bidder and the shareholders, are higher in  $\mathcal{E}_1$ . This means that everyone is always better off, if some bidder type restricts her bid more and  $q(\cdot)$  is adjusted accordingly. Choosing r = 0.5 enforces  $\gamma = 0.5$ , thereby minimizing the fraction of shares purchased in the event of success.

## **Proof of Proposition 5**

The proof is mainly in the text. It remains to show that, if  $\phi'(\cdot) > 0$ , the first-order condition is sufficient to identify  $V = \hat{V}$  as the unique solution to the bidder's maximization problem. Differentiating the bidder's objective function yields

$$\partial \Pi / \partial \hat{V} = 0.5 \phi'(\hat{V}) V - P'(\hat{V}).$$

If the functions  $\phi(\cdot)$  and  $P(\cdot)$  satisfy the first-order condition, we know that  $0.5\phi'(\hat{V})\hat{V} = P'(\hat{V})$ . Substituting this equation into the derivative gives

$$\partial \Pi / \partial \hat{V} = 0.5 \phi'(\hat{V})(V - \hat{V}).$$

When  $\phi'(\cdot) > 0$ , this implies  $\partial \Pi / \partial \hat{V} > 0$  for all  $\hat{V} < V$  and  $\partial \Pi / \partial \hat{V} < 0$  for all  $\hat{V} > V$ . That is, the objective function is quasi-concave. Given that the bidder's choice set  $\mathcal{V}$  is compact and convex, it follows that the bidder's objective function has a unique global maximum at  $\hat{V} = V$ .

## **Proof of Proposition 6**

Properties of the equilibrium schedule. A fully revealing equilibrium requires  $t'(\cdot) < 0$ . This follows from the non-mimicking constraints for type X and x, which are respectively

$$C(X) - [1 - t(X)]H(X) \le C(x) - [1 - t(x)]H(X)$$
  
and  
 $C(x) - [1 - t(x)]H(x) \le C(X) - [1 - t(X)]H(x).$ 

For t(X) = t(x), the inequalities hold jointly only if C(X) = C(x), in which case the two offers would be identical. For t(X) > t(x), rewrite the inequalities as

$$C(x) \ge C(X) + [t(X) - t(x)]H(X)$$
 and  $C(x) \le C(X) + [t(X) - t(x)]H(x)$ 

Since C(X) + [t(X) - t(x)]H(X) > C(X) + [t(X) - t(x)]H(x), the constraints cannot hold jointly. Thus, the non-mimicking constraints are violated unless  $t(\cdot)$  is decreasing.

In addition, the first-order condition

$$-t'(X)H(X) - C'(X) = 0$$
(16)

must be satisfied. Similar calculations as in the previous proof show that condition (16) and  $t'(\cdot) < 0$  imply quasi-concavity of the objective function. That is, they are jointly necessary and sufficient for incentive-compatibility. Furthermore, they imply  $C'(\cdot) > 0$ .

Relative bid premia. Condition (16) and  $t'(\cdot) < 0$  have implications for the relative bid premium  $\Delta \equiv t(X)H(X) + C(X) - X$ . In particular,  $\partial \Delta / \partial X = t(X)(1+\lambda) - 1$  and  $\partial^2 \Delta / \partial X^2 < 0$ . The first derivative implies that the premium *increases* with the bidder's type when  $t(X) > 1/(1+\lambda)$ , while the second derivative implies that  $\partial \Delta / \partial X$  is strictly decreasing. For instance, if  $t(\overline{X}) > 1/(1+\lambda)$ , the bid premium is increasing at  $\overline{X}$ , and must also be increasing at all types  $X \leq \overline{X}$ . That is, lower types then pay a *smaller* premium.

If  $t(\overline{X})$  is so high that type  $\overline{X}$  is a net issuer, it follows from  $t'(\cdot) < 0$  that all lower types are also net issuers and hence have an incentive to mimic type  $\overline{X}$ . To reveal their lower type, they must be given information rents in the form of smaller bid premia than  $\overline{X}$ . However, type  $\overline{X}$  cannot be forced to pay a positive premium, as she can always succeed with the offer  $(C, t) = (\overline{X}, 0)$ . Lower types will therefore mimic  $\overline{X}$  unless they pay negative premia, which violates the free-rider condition. Thus, unless type  $\overline{X}$  is a net purchaser, there exists no fully revealing equilibrium.

If type  $\overline{X}$  is a net purchaser, bid premia are increasing for all lower types that are also net purchasers but are decreasing for those types that are net issuers. The bid premia may thus turn negative below some type  $X_{\Delta}^c$ . But this requires  $t(X_{\Delta}^c)(1 + \lambda) > 1$  and thereby  $t(X_{\Delta}^c)H(X_{\Delta}^c) > X_{\Delta}^c$ . The latter inequality implies  $C(X_{\Delta}^c) < 0$ , as the free-rider condition is binding for type  $X_{\Delta}^c$ . By continuity of  $C(\cdot)$ , some higher types must also pay a negative cash price, which means that the cash constraint always becomes binding before incentive-compatible bid premia become negative.

*Equilibrium profits.* As before, we can use the envelope theorem to determine at which rate takeover profits must vary in equilibrium:

$$\partial \Pi^* / \partial V = [1 - t(X)](1 + \lambda).$$

Given that type  $\overline{X}$  pays a zero premium, her takeover profit is S. As profits decrease at the above rate, there may exist a type  $X^c_{\pi}(S) \in [0, \overline{X})$ , defined by

$$\int_{X_{\pi}^{c}(S)}^{\overline{X}} \{ [1 - t(u)](1 + \lambda) \} du = S,$$
(17)

below which profits become negative. Larger  $t(\cdot)$  makes it less attractive for high types to mimic low types, thereby mitigating the asymmetric information problem. As a result, the rate at which profits must decrease to deter mimicking are smaller, so that  $X_{\pi}^{c}$  is lower. By increasing the bidder's willingness to pay premia, a larger S also reduces  $X_{\pi}^{c}$ . Finally, for a given  $t(\cdot)$ , a larger  $\lambda$  increases the value of the equity component that the bidder gives away, thus accelerating the decrease in profits. From the discussion about bid premia above, we know that profits must be decreasing faster than under full information at least for some interval below type  $\overline{X}$ .

Cut-off type.  $X_{\pi}^c$  is a potentially binding cut-off type, which reflects the bidder's participation constraint. Similar cut-off candidates can be derived from the control constraint  $t \in [0, 0.5]$  and the cash constraint  $C \geq 0$ . For a given  $t(\cdot)$ , it is straightforward to see that there may exist a type  $X_t^c \in [0, \overline{X})$ , defined by  $-\int_{X_t^c}^{\overline{X}} t'(u) du = 0.5 - t(\overline{X})$ , such that t(X) > 0.5 for all lower types. The control constraint is stricter for higher  $|t'(\cdot)|$  and higher  $t(\overline{X})$ . In case of the cash constraint, type  $\overline{X}$ 's cash component must satisfy  $C(\overline{X}) = \overline{X} - t(\overline{X})A(\overline{X}) \geq 0$ , as she pays a zero premium. Hence, for a given  $t(\cdot)$ , there may exist a type  $X_C^c \in [0, \overline{X})$ , defined by  $-\int_{X_C^c}^{\overline{X}} t'(u)H(u)du = C(\overline{X})$ , below which cash prices become negative. The constraint is stricter, if bidder assets are larger (high  $\lambda$  or Z), or if the equity component grows faster (high  $|t'(\cdot)|$ ).

Which of the three constraints (participation, cash or control) is binding – and hence which among  $X_{\pi}^{c}$ ,  $X_{t}^{c}$  and  $X_{C}^{c}$  is the true cut-off type – depends on the chosen schedule that satisfies condition (16). Interestingly, there is a trade-off. From the definition (17), we know that  $X^c_{\pi}$  is lower if |t'(X)| or  $t(\overline{X})$  are larger. However, by issuing more holding company shares to the shareholders, the bidder is more prone to violate the cash constraint or the control constraint. Contrary to  $X^c_{\pi}$ , the other cut-off candidates  $X^c_C$  and  $X^c_t$  are therefore higher if |t'(X)| or  $t(\overline{X})$  are larger. The most efficient schedule is hence reached by increasing  $t(\cdot)$  until  $X^c_{\pi}$  and max  $\{X^c_C, X^c_t\}$  are equal. For instance, simple inspection of its definition shows that  $X^c_C$  ceteris paribus increases for larger Z or  $\lambda$ , as H(u) becomes larger and  $C(\overline{X})$  becomes smaller. While  $X^c_C$  can in response be reduced again by decreasing either  $t(\overline{X})$  or |t'(X)|, this would in turn increase  $X^c_{\pi}$ , as can be seen in its definition (17).  $C \geq 0$  is thus a more severe signaling constraint for large bidders.

# Appendix A: Pooling Equilibria

We first focus on the deterministic setting. In a pooling equilibrium, the equilibrium bid does not fully resolve the uncertainty about the bidder's type. That is, a given type either makes no bid or chooses offer terms that are also chosen by other types. It is individually rational for a bidder (type) to submit this offer if and only if  $\Phi(X) + r(X - P) \ge 0$ . For a given offer (r, P), let  $\mathcal{X}_{r,P} \subseteq \mathcal{X}$  denote the set of bidder types for whom the participation constraint is satisfied.

The equilibrium bid must also satisfy the free-rider condition  $P \ge E(X|X \in \mathcal{X}_{r,P})$ . There exists a continuum of prices that satisfy this condition, and so constitute Perfect Bayesian Equilibria of the tender offer game. Following Shleifer and Vishny (1986), we select the minimum bid equilibrium which is the unique equilibrium satisfying the credible beliefs criterion of Grossman and Perry (1986). All other equilibria require shareholders to believe that bidders generate, on average, security benefits that are smaller than the offered equilibrium price. Imposing the credible beliefs criterion implies that shareholders do not reject a bid consistent with the free-rider condition, and that bidders hence choose the smallest price such that the condition is satisfied.

**Proposition 9** The deterministic tender offer game with exogenous private benefits has a unique Perfect Sequential Equilibrium. All types  $X \in \mathcal{X}_{0.5, P_{\min}^*(0.5)}$  make a bid for half the shares offering the same price  $P_{\min}^*(0.5)$ , which is the smallest price such that  $P \geq E(X | X \in \mathcal{X}_{0.5, P}).$ 

**Proof.** By definition, every  $X \in \mathcal{X}_{r,P}^c$  satisfies the participation constraint  $\Phi(X) + r(X - P) \ge 0$  which can be rewritten as

$$X \ge P - \frac{\Phi(X)}{r}.$$
(18)

For expositional convenience, define the function  $g(r, P) \equiv \mathbb{E}(X | X \in \mathcal{X}_{r,P}^c)$ . Note that  $g(r, 0) = \mathbb{E}(X) > 0$ , whereas  $g(r, \overline{X}) = \mathbb{E}(X | X \ge \overline{X} - \Phi(X) / r) < \overline{X}$ . That is, the point (0, g(r, 0)) lies above the 45°-line in the P-g(r, P)-space, whereas  $(\overline{X}, g(r, \overline{X}))$  lies below the 45°-line. The latter implies that, for any r, there exists a set of prices  $\mathcal{P}(r) \subset \mathcal{X}$  which satisfy the free-rider condition. (Moreover, if g(r, P) is continuous in P, it also implies that there exists at least one fixed point.)

In addition to satisfying the free-rider condition, any equilibrium pooling offer (r, P)must yield at least as high (positive) profits as any alternative bid  $(r^o, P^o)$ . Any bid  $(r^o, P^o)$  with  $P^o < \overline{X}$  can be made to fail by choosing shareholders' off-equilibrium beliefs  $E[X | (r^o, P^o)] = \overline{X}$ , and can hence be ruled out as a profitable deviation. All bids  $(r^o, P^o)$  with  $P \ge \overline{X}$  always succeed, irrespective of shareholders' off-equilibrium beliefs. Among these offers, the least costly, and hence most profitable, one is  $(0.5, \overline{X})$ . Hence, only offers that belong to  $\mathcal{P}(r)$  and satisfy the condition

$$\max\{r(X-P), 0\} \ge 0.5(X-\overline{X})$$

for all  $X \in \mathcal{X}$  can be supported as Perfect Baysian Equilibria. Let  $\mathcal{P}^*(r)$  denote the set of equilibrium pooling prices for a given r. As the offer  $(0.5, \overline{X})$  satisfies both conditions, it belongs to  $\mathcal{P}^*(0.5)$ .<sup>21</sup>

If  $\mathcal{P}^*(r)$  is non-empty, it typically has multiple elements. To select a unique equilibrium pooling price for a given  $r \in [0.5, 1]$ , we use the credible beliefs criterion. Denote as  $\mathcal{D}(P^o, P; r) \subseteq \mathcal{X}$  the set of types that would prefer the deviation price  $P^o$  over the (conjectured) equilibrium price P, given the deviation offer were to succeed. Observing a deviation  $P^o$ , shareholders are said to have credible beliefs if they attribute this deviation only to types from  $\mathcal{D}(P^o, P; r)$ , i.e.  $\mathbb{E}[X|X \in \mathcal{D}(P^o, P; r)]$ . If, based on such beliefs, shareholders accept the bid  $P^o$ , then the Perfect Bayesian Equilibrium price P cannot be supported by credible beliefs, and hence is not a Perfect Sequential Equilibrium.

Now consider an equilibrium price  $P(r) > P_{\min}^*(r)$ . Every type  $X \in \mathcal{X}_{r,P_{\min}^*(r)}^c$  would deviate to  $P_{\min}^*(r)$ , if this bid were to succeed. This trivially holds for any type whose participation constraint is violated under P(r) but not under  $P_{\min}^*(r)$ . It also holds for all  $X \in \mathcal{X}_{r,P(r)}^c$ , as any bidder *ceteris paribus* prefers to succeed at a lower price. Since  $\mathcal{D}(P_{\min}^*(r), P(r); r) = \mathcal{X}_{r,P_{\min}^*(r)}^c$ , credible beliefs following a deviation to  $P_{\min}^*(r)$  are therefore restricted to  $g(P_{\min}^*(r), r)$ . Since  $P_{\min}^*(r) \in \mathcal{P}^*(r)$ , the free-rider condition is satisfied for those beliefs, and the deviation would succeed. As a result, only  $P_{\min}^*(r)$  can be supported as a Perfect Sequential Equilibrium.

Similarly, any Perfect Bayesian Equilibrium offer (r, P) with r > 0.5 is not robust to deviations to (0.5, P) under credible beliefs. That is, for a given price, only r = 0.5

<sup>&</sup>lt;sup>21</sup>Depending on the distribution of private benefits across types, there may exist a continuum of degenerate pooling equilibria  $(r, \overline{X})$  with  $r \in [0.5, 1]$ , in which only type  $\overline{X}$  makes a bid. In this case, the credible beliefs criterion does not select a unique equilibrium.

can be a Perfect Sequential Equilibrium. Consider a equilibrium pooling offer (r, P) with r > 0.5. Every type  $X \in (P, \overline{X}]$  acquires shares below their true value, whereas every type  $X \in [0, P)$  either pays a premium or abstains from bidding. Hence, any type that deviates from (r, P) to (0.5, P) must have security benefits below P. Given shareholders hold credible beliefs, E[X|(0.5, P)] < P. As the free-rider condition is strictly satisfied, the deviation offer would succeed. As a result, only (0.5, P) can be supported as a Perfect Sequential Equilibrium.

Given the credible beliefs criterion rules out any  $P(r) > P_{\min}^*(r)$  and any r > 0.5, the only "true" Perfect Sequential Equilibrium is  $(0, 5, P_{\min}^*(0.5))$ .

The equilibrium bid is a noisy signal, which only reveals that the bidder belongs to the subset of types who profit from this bid. Among the successful bidder types, some are overvalued and some are undervalued. Nevertheless, the price equals the expected security benefits for that subset, so that on average shareholders extract the full posttakeover share value. For some types, the mispricing may be so severe that a takeover at that price is unprofitable. Moreover, these bidder types cannot succeed with a lower offer because all higher types would then make the same offer, and target shareholders would on average be offered less than the post-takeover security benefits. It should be noted that the equilibrium outcome is efficient ( $\mathcal{X}_{0.5,P_p} = \mathcal{X}$ ) if the private gains are sufficiently large for every type, but it is completely inefficient ( $\mathcal{X}_{0.5,P_p} = \{\overline{X}\}$ ) in the absence of private gains. Depending on the distribution of private benefits  $\Phi(\cdot)$ , the set  $\mathcal{X}_{0.5,P_p}$  need not be convex. For instance, there need not exist a threshold type such that all and only types above the threshold make a bid.

Although our setup assumes a general  $\Phi(\cdot)$ -function and treats r as a choice variable of the bidder, the equilibrium has the same properties as the minimum bid equilibrium in Shleifer and Vishny (1986). Specifically, the equilibrium price is the smallest price that satisfies the free-rider condition, and bids are fully restricted. The latter property follows from the equilibrium refinement. Overvalued types prefer to buy fewer rather than more shares, where the opposite holds true for undervalued types. Under credible beliefs, shareholders therefore attribute any deviation bid that is more restricted than the proposed equilibrium bid to an overvalued bidder, and hence accept it. Only the fully restricted bid itself is robust to such deviations. Incidentally, it minimizes the redistribution from low to high types, thereby making bids more profitable for the former.<sup>22</sup>

The deterministic pooling equilibrium can be replicated with probabilistic outcomes for some, but not all, q(r, P) < 1. Suppose that a given pooling offer (r, P) succeeds with probability q(r, P). For a given type, submitting this offer is individually rational if and only if  $q(r, P) [\Phi(X) + r(X - P)] \ge 0$ . The sign of the left-hand side is independent

 $<sup>^{22}</sup>$ As K = 0, this also maximizes the range of successful types, i.e. takeover probability. When K > 0, this is not necessarily the case as higher types may have insufficient private benefits to cover K, and hence rely on the gains from the purchased shares to make a profitable bid (Marquez and Yilmaz, 2005).

of the takeover probability, so that changes in q(r, P) leave the set of types for whom the participation constraint is satisfied, and hence shareholders' expectations about the post-takeover share value conditional on a bid, unaffected. Thus, if a pooling equilibrium offer can be supported under probabilistic outcomes, it can also be supported under deterministic outcomes. Given that all bidder types are value-improving, the probabilistic pooling outcomes are Pareto-dominated by the corresponding deterministic outcome.

# Appendix B

Suppose now that a constant  $\gamma(P) = \gamma$  cannot ensure that the left-hand side in (14) changes sign only once, namely at P = X. In this case, the bidder's objective function is no longer quasi-concave, and the first-order condition may not identify a global solution to the maximization problem, for r = 0.5. Returning to the more general case, note that the basic principle for making  $\Pi(P; X)$  quasi-concave is to put more weight on the second term on the left-hand side of (14), thereby giving it more weight in determining the sign of the derivative. Since a constant  $\gamma(P) = \gamma$  can at most give A a value of 1, the question is whether  $\gamma(\cdot)$  can be chosen such that A > 1 when needed. In principle, this is possible since A depends not only on the level but also the *slope* of  $\gamma(\cdot)$ . In fact,

$$\gamma(P) + \frac{\gamma'(P)}{\gamma(P)} \Phi(P) > 1 \Leftrightarrow \gamma'(P) > \frac{\gamma(P) \left[1 - \gamma(P)\right]}{\Phi(P)} > 0.$$

Thus, alleviating the bid restriction for certain bidder types may "iron out" the objective function such that it is quasi-concave. Note that, *in* equilibrium, it is pay-off irrelevant how many shares the bidder acquires. However, these differences may be necessary to satisfy all non-mimicking constraints, that is, to *support* the equilibrium.

This potential solution has two caveats. First, the required increase in  $\gamma(\cdot)$  may violate the boundary constraints  $\gamma(\cdot) \in [0.5, 1]$ . Second, if the incentive-compatible  $\gamma(\overline{X})$ exceeds 0.5, the equilibrium cannot be supported by out-of-equilibrium beliefs. In either case, the intuition is that the downstream and the upstream non-mimicking constraints cannot be reconciled with each other. More specifically, the decrease in takeover probability needed to prevent high types from mimicking lower prices is so severe that, conversely, low types prefer to mimic the high offers to "save" their private benefits. Instead of thoroughly analyzing the (in our view, less plausible) case when (a) and (b) are violated, we provide a discrete-type example.

**Example.** Consider two types,  $i \in \{H, L\}$ . Their security benefits are given by  $X^H$  and  $X^L$ , where  $\Delta_X \equiv X^H - X^L > 0$ . The corresponding private benefits are given by  $\Phi^H$  and  $\Phi^L$ , with  $\Delta_{\Phi} \equiv \Phi^H - \Phi^L < 0$ . Conjecture a fully revealing equilibrium, in which each type bids her post-takeover share value. Let  $q^i$  and  $\gamma^i$  respectively denote the takeover probability and the expected fraction of tendered shares associated with *i*'s equilibrium

offer.

From the proof of Proposition 2, we know that H's offer must succeed with certainty. Thus,  $q^H = 1$ . This provided, the maximum  $q^L$  such that H does not mimic L is given by

$$\Phi^{H} = q_{\max}^{L} \left( \Phi^{H} + \gamma^{L} \Delta_{X} \right) \quad \Leftrightarrow \quad q_{\max}^{L} = \frac{\Phi^{H}}{\Phi^{H} + y^{L} \Delta_{X}}$$

Note that  $q_{\text{max}}^L$  is decreasing in  $y^L$ . By the same arguments as in Step 1 of the proof of Proposition 2, the Pareto-dominance criterion thus requires  $y^L = 0.5$ , which is achieved by setting  $r^L = 0.5$ .

Conversely, the minimum  $\gamma^H$  such that L does not mimic H is given by

$$q^{L}\Phi^{L} = \Phi^{L} - \gamma^{H}_{\min}\Delta_{X} \quad \Leftrightarrow \quad \gamma^{H}_{\min} = \left(1 - q^{L}\right)\frac{\Phi^{L}}{\Delta_{X}}.$$

Note that  $\gamma_{\min}^{H}$  is decreasing in  $q^{L}$ . Hence, it is minimized for  $q^{L} = q_{\max}^{L}$  (with  $y^{L} = 0.5$ ). Substituting and simplifying yields

$$\gamma_{\min}^{H} = \frac{0.5\Phi^{L}}{\Phi^{H} + 0.5\Delta_{X}}$$

We can now make three observations: (i) It is straightforward to verify that, if the analogue of condition (b),  $|\Delta_{\Phi}| > 0.5 |\Delta_X|$ , is violated, then it must be that  $\gamma_{\min}^H > 0.5$ . That is, H and L must differ in their bid restriction to satisfy incentive-compatibility. (ii) As regards the first caveat, note that, for any  $\Delta_X$  and  $\Phi^H$ , there exist sufficiently large  $\Phi^L$  such that  $\gamma_{\min}^H > 1$ . Since this is not feasible, separation is then impossible. (iii) Irrespective of the previous observation, note that separation cannot be sustained even if  $\gamma_{\min}^H \in (0.5, 1]$ . Indeed,  $\gamma_{\min}^H > 0.5$  implies that there always exists an offer  $(0.5, \overline{X} + \epsilon)$ , where  $0 < \epsilon \ll 1$ , that succeeds with certainty and is preferred by L over her own equilibrium offer. Thus, in this two-type example, the second caveat makes the first caveat irrelevant, as no  $\gamma_{\min}^H$  larger than 0.5 can be sustained as an equilibrium. This need not be the case when there are more than two types. In that case, the first caveat may apply to interior types even when a maximally restricted offer for the highest type is incentive-compatible.

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